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A MANUAL

OF

ARITHMETIC.

BY THE

REV. CHARLES WHITE UNDERWOOD, M.A.
VICE-PRINCIPAL OF LIVERPOOL COLLEGE;
LATE SCHOLAR OF ST. JOHN'S COLLEGE, CAMBRIDGE.

NEW EDITION.

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PREFACE

TO

THE SECOND EDITION.

THE WORK has been very carefully revised, mistakes in the printing have been rectified, new methods of working examples in Compound Interest and Cube Root have been substituted, a chapter on the Exchange of Money, and one, containing the rules and formulæ of Mensuration, have been added to the letter-press.

Nearly one thousand five hundred examples are given at the end of the book in a series of papers. It will be seen, on inspection, that they are so arranged as to form exercises for the student as he advances, and also to supply Examination papers on the whole or any part of the subject. Three examples are given to each Rule, increasing in difficulty.

The book has been adopted as the text-book on Arithmetic in the Schools of the Liverpool College. From the success attending its use, I have been encouraged to send it forth in a more complete and useful form; and again I have to acknowledge the kindness of my friends, Mr. W. Finlay and Mr. J. Garnett, to both of whom I am greatly indebted for much valuable assistance.

The Answers to the Examples are published separately. Owing to the hurry and pressure of school-work, a few questions scattered here and there in the papers have escaped revision. An explanation of the ambiguity will be found in the answers, or in the errata prefixed.

C. W. UNDERWOOD.

LIVERPOOL COLLEGE.

PREFACE

TO

THE FIRST EDITION.

So MANY complete treatises on Arithmetic have appeared within the last few years, that it seems something like presumption to send forth another *small* manual.

The object aimed at by the compiler is not, however, to add anything particularly new or original, but to bring before *junior* students so much of the Theory of Arithmetic as may be fairly expected of them, and to present it in such a form that the study of the science may become to some extent a *mental training.*

It is rather a *Grammar* of Arithmetic than a treatise on that subject, and should, for the most part, be committed to memory like the rudiments of a language.

It will be found well adapted for *vivâ voce* examination, and will afford full scope, to an intelligent teacher, for lessons in "*Mental Arithmetic*," rightly so called.

One other object of the work is to enable candidates to prepare themselves for the Local University Examinations, in as far as the Science of Arithmetic is concerned.

The *Definitions* are briefly and (it is hoped) carefully worded. Each rule is stated so as to include the proof of it, where this was possible.

Except in the cases of the "Square Root" and "Cube Root" no algebraical expression is employed.

The *Practice* of Arithmetic is exemplified so as to exercise the judgment. When the Student has once mastered the notion of "*the Unit*," which is constantly occurring in the Examples given at the end, he will be independent of rules, and will find no difficulty in working out any question which may be proposed to him.

I gratefully acknowledge here the valuable assistance rendered me by my friend W. Finlay, Esq., Mathematical Master of the Middle School of the Liverpool Collegiate Institution.

C. W. UNDERWOOD.

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CHAPTER I.

DEFINITIONS.

1. *Arithmetic* is the science of numbers.

2. There are ten figures or symbols in use, for the purpose of Arithmetical Calculation, viz. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, called respectively, *zero* or *cipher*, *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*. All but the first are also called digits.

They were introduced into Europe by the Saracens, probably from the Hindoos.

3. *A Unit* is one of any kind.

4. *A Number* is a collection of units of the same kind.

5. *An Abstract Number* has no substantive to qualify it.

6. *A Concrete Number* defines the kind of unit; thus, *four* is Abstract, but *four men* is Concrete.

7. The nine digits have two values each :—

(i.) *An Intrinsic value*, or that which they represent when standing alone.

(ii.) *A Local value*, or that which they acquire by being placed before one or more others.

8. The terms used to facilitate computation are *unit*, *ten*, *hundred*, *thousand*, *million*, *billion*, &c.; and combinations of these, as thirteen, *i.e. three and ten;* twenty, *i.e. two tens*, &c.

9. *Notation* is the art of expressing numbers by means of figures.

10. *Numeration* is the art of interpreting numbers already expressed in figures.

11. *The radix of Notation* is the number which enables us to fix the local values of the digits. The radix may be any number except 1, but in the ordinary scale it is *ten.*

12. *The Numeration Table* (with radix *ten*) is—

Unit.
Ten, *i.e.* ten units, or the radix.
Hundred „ tens, or ten times the radix.
THOUSAND „ hundreds, or hundred times the radix.
Ten thousands.
Hundred thousands.
MILLION.
Ten Millions.
Hundred Millions.
Thousand Millions.
Ten thousand Millions.
Hundred thousand Millions.
BILLION.
&c.

N.B.—Of any one of these various *denominations* no higher number than nine can be expressed by one figure.

13. *The Notation Table* is—

1.
10.
100.
1000.
10,000.
100,000.
1000,000.
10,000000.
100,000000.
1000,000000.
10000,000000.
100000,000000.
1000000,000000.
&c.

14. From the preceding tables we see that the *local value* of the digits depends on the following rule (the radix being *ten*) :—

Any digit followed by one other is increased tenfold.

Any digit followed by two others is increased a hundredfold.

Any digit followed by three others is increased a thousandfold.

And so on.

Thus we count units up to *ten;* tens and units up to *one hundred;* hundreds, tens, and units up to *one thousand;* and so on.

Further: the first figure on the right is styled the units' figure.

The second figure on the right is styled the tens' figure.

The third figure on the right is styled the hundreds' figure.

And so on as below :—

Billions.	Hundred thousand millions.	Ten thousand millions.	Thousand millions.	Hundred millions.	Ten millions.	Millions.	Hundred thousands.	Ten thousands.	Thousands.	Hundreds.	Tens.	Units.
1	0	0	0	0	0	0	0	0	0	0	0	0

15. (*a*) The Roman letters, I, V, X, L, C, D, M, are used chiefly for marking chapters of books or volumes, or the superior divisions of a treatise; for the date of the years, and in inscriptions, &c.

I	represents	intrinsically	*one*	unit.
V	„	„	*five*	units.
X	„	„	*ten*	„
L	„	„	*fifty*	„
C	„	„	*hundred*	„
D or IↃ	„	„	*five hundred*	„
M or CIↃ	„	„	*one thousand*	„

(*β*) A letter placed before one of a *superior* value subtracts its value from the *superior*.

(γ) A letter placed after one of a *superior* value adds its value to the *superior*.

(δ) *The same letter* repeated *once*, *twice*, &c. *doubles*, *trebles*, &c. its original value; *e.g.* XXX represents three times X, or 30.

(ε) Every Ɔ annexed to IƆ increases the latter tenfold; *e.g.* IƆƆ denotes 5000.

(ζ) Every C prefixed and Ɔ annexed to CIƆ increase the latter tenfold; *e.g.* CCIƆƆ denotes 10000.

(η) *A bar*, or *vinculum*, placed over a letter, multiplies it by one thousand; *e.g.* $\overline{\text{M}}$ denotes one thousand thousands.

16. *Addition* is the method of finding the amount of various collections of units of the same kind.

17. *The Sum* is the number so found.

18. *Subtraction* is the method of finding by how many units, tens, hundreds, &c. a greater number exceeds a less.

19. The greater is sometimes called the *minuend;* the less, the *subtrahend.*

20. *A Multiple* is a number which contains another an exact number of times.

21. *A Measure* is a number which is contained an exact number of times in another.

Thus, 20 is a multiple of 5, and 5 is a measure of 20.

22. *Multiplication* is a short method of Addition of

equal numbers, depending upon the ready knowledge of the simple *multiples* of the digits.

23. *A Factor* is a multiplier which helps to make a product, or it is one of the measures of a number; thus, 3, 4, 5 are factors, which, multiplied together, make 60; and, conversely, 60 may be split up into the factors 3, 4, 5.

24. *The product* of two numbers is the amount which is obtained by multiplying them together. It is called the *continued product* when more than two numbers are multiplied together.

25. *Division* is the method of finding how many times one number measures another; or of finding a number which will be a given measure of another given number; or it is a method of finding one of the factors of a given product when the other is given.

26. *The Quotient* is the result arising from the division of one number by another.

27. *A Composite Number* is that which is made up of factors. *e.g.* 24, 40, 48 are composite numbers.

N.B.—1 is not a factor.

28. *A Prime Number* is that which has no factor.

e.g. 3, 7, 11, 13 are prime numbers.

Numbers are said to be prime to one another when they have no common factor.

29. *A Common Measure* of two or more numbers is a measure of each of them; or, it is any factor or product of factors common to them all.

30. *The Greatest Common Measure* of two or more

numbers is the greatest factor or product of factors common to them all.

This is sometimes called the *Highest Common Divisor.*

31. *An Aliquot Part* is one of the equal parts into which a whole is divided.

32. *A Common Multiple* of any set of numbers is a multiple of each of them.

33. *The Least Common Multiple* of any set of numbers is the least number which contains each of them without a remainder.

34. *Incommensurable Numbers* are those which have no common measure.

35. *Linear Measure* is the measurement of lines or distances by means of lines of fixed standard length.

36. *Superficial Measure* is the measurement of areas or superficies by means of areas of fixed standard dimensions.

37. *Solid* or *Cubic Measure* is the measurement of solids, volumes, or contents, by means of solids of fixed standard dimensions.

38. There may be an infinite number of units of measure, but certain standards have been fixed on by Government, which are the only ones recognised.

39. *The Imperial Standard Yard* of 36 inches was originally made of brass very accurately measured. It was destroyed in the fire of the House of Commons in 1834, together with the other standards of weight and measure.

Its exact length, however, is recoverable by means of a pendulum, which vibrates in one second in lat. 51½°, in a vacuum, and at the level of the sea. Such a pendulum is found to be 39·1393 inches long.

40. *The Weight* of a body is properly the attraction of gravitation on it, and is measured by the support required to prevent its falling to the earth.

41. *The Standard Pound Troy* is made of brass, and contains 5760 grains. The pound avoirdupois contains 7000 grains.

Since a cubic inch of distilled water weighs 252·458 grains Troy (the mercury in the barometer standing at 30 inches, and the thermometer at 62), the weight of the imperial standard pound may be always compared or recovered.

42. *The Imperial Standard Gallon* contains 10 lbs. of distilled water under the same conditions of temperature and atmospheric pressure, and forms a volume of 277·274 cubic inches.

43. *The Imperial Bushel* contains 8 gallons of such distilled water, or 2218·192 cubic inches.

44. *Standard Gold* contains 22 parts of pure gold to 2 parts of alloy.

45. *Standard Silver* contains 37 parts of pure silver to 3 parts of copper.

46. *Jewellers' Gold* contains only 18 parts of pure gold out of 24.

47. *Length* is the distance to which anything extends.

48. *Breadth*, or *Width*, is the distance to which a surface expands.

49. *Depth* is the distance to which a solid extends, measured directly downwards from the surface.

That which has only length is said to be of one dimension. That which has length and breadth is said to be of two dimensions. That which has length, breadth, and depth is said to be of three dimensions.

50. *Reduction* is the method of expressing the higher denominations of concrete numbers in terms of all the lower denominations, or of any one of them, and the converse.

51. *Practice* is the method of finding the value of inferior denominations of concrete numbers, from knowing the value of one of the higher denominations of that kind.

52. The following signs or symbols are used to facilitate arithmetical operations:

+, called '*plus*,' placed between two numbers signifies that the *latter* is to be added to the former.

−, called '*minus*,' placed between two numbers signifies that the one before which it is placed is to be subtracted from the other.

×, '*into*,' placed between two numbers signifies that the one before which it is placed is to multiply the other.

÷ placed between two numbers signifies that the one before which it is placed is to divide the other.

=, '*equal*,' signifies that the numbers or expressions between which it is placed are equal.

[{()}] Each pair of similar forms is called a bracket, and signifies that all the quantities inclosed within it are affected in the same way.

——, called a ***vinculum***. This bar placed over any number of quantities has the effect of a bracket: *e. g.* $\overline{2\frac{1}{2} - \frac{3}{4\frac{1}{2}} + 6}$ is the same as $\left(2\frac{1}{2} - \frac{3}{4\frac{1}{2}} + 6\right)$.

$>$ signifies '*greater than*.'

$<$ „ '*less than*.'

$\propto$ signifies that the quantity placed before it varies as the one after it.

53. *An Integer* is a whole number.

54. *A Fraction* is a part, or parts, of an integer.

65. *A Proper Fraction* is less than an integer.

56. *An Improper Fraction* is a conventional term, implying that one or more whole numbers of the same kind and value have each been divided into the same number of equal parts, and that so many of these parts are taken as would make a whole, or more than a whole.

57. *A Compound Fraction* is a fraction of a fraction.

58. *A Mixed Number* is a whole number and a fraction.

59. There are two parts to a fraction.

(i.) *The Denominator*, which denotes the number of equal parts into which the whole has been divided.

(ii.) *The Numerator*, which denotes the number of those parts that are taken.

A fraction is expressed thus: $\frac{\text{Numerator}}{\text{Denominator}}$, and is said to be in its lowest terms when the numerator and denominator are prime to each other.

60. *A Complex Fraction* contains a mixed number or a fraction in the numerator or denominator, or in both.

61. *Vulgar Fractions* are those which can have any denominator.

62. *Decimal Fractions* are those which have ten or some power of 10 for their denominator.

63. *The Decimal Point* is a point which, by its position, indicates the denominator, and saves the necessity of expressing it.

If the point is immediately after the tens' figure, the denominator is 10.

If the point is immediately after the hundreds' figure, the denominator is 100; and so on.

Thus in 234·5, the denominator is ten, and the fraction may be written $\frac{2345}{10}$.

In 23·45, the denominator is 100, and the fraction may be written $\frac{2345}{100}$.

64. *A recurring Decimal*, called also a *circulating* or *repeating Decimal*, is that in which the same figure or figures continually recur, and in the same order.

65. *A pure recurring Decimal* is that in which the whole decimal part recurs in the same order.

66. *A mixed recurring Decimal* is that in which only a part of it recurs.

67. *The Repetend* or *Period* is the figure or sequence of figures that recurs. A point is placed over the figure which recurs, or over the first and last of the figures composing the Period.

e. g. 23·444 . . . . . is represented by $23{\cdot}\dot{4}$;
and 23·456456 . . . „ „ $23{\cdot}\dot{4}5\dot{6}$.

4 is the *period* of the first, and 456 of the second.

68. *A Power of a Number* is the continued product of it by itself; the number of factors being equal to the number denoted by the power.

69. *Involution* is the method of finding any assigned power of a given number.

70. *The Root* of a number is some number which, when raised to the power denoted by the root, will produce the given number.

71. *Evolution* is the method of finding any assigned root of a given number.

72. *The Index* or *Exponent* is the number denoting the power or the root, and is placed at the top of the

number to the right:—thus, in 4^5, ^{5} is the index or exponent.

73. *The Symbol of Evolution* is $\sqrt{\ }$; a number denoting the root being placed in the angle: *e. g.* $\sqrt[3]{\ }$ denotes the Cube Root.

74. *A Surd* is a quantity expressed with the sign of evolution, but of which the root denoted by the sign cannot be found exactly.

75. *Positive Numbers* are those which are not affected by a sign.

76. *Negative Numbers* are those which have the sign $-$ before them.

77. *Impossible Quantities* are surd expressions denoting the even roots of negative numbers which cannot be determined: thus, $\sqrt{(-2)}$, $\sqrt[4]{-a^4}$ are impossible quantities.

78. The powers of all positive quantities have a positive result.

79. The *odd* powers of negative quantities have a negative result.

80. The *odd* roots of negative quantities are negative.

81. *Ratio* is the relation which two quantities of the same kind bear to one another, in respect of magnitude; or it is the multiple, part, or parts which one quantity is of another of the same kind.

: is the symbol used to connect the two quantities: thus, $a : b$, $2 : 4$, express the *ratio* of a to b, and 2 to 4.

a and b, 2 and 4, are respectively called the terms of the ratio.

82. *The Antecedent* is the first member or term of a ratio.

83. *The Consequent* is the second member of a ratio. The fraction, $\frac{\text{Antecedent}}{\text{Consequent}}$ expresses the ratio.

84. *A Ratio of Equality* is that in which both terms are equal.

85. *A Ratio of greater inequality* is that in which the first term is greater than the second.

86. *A Ratio of less inequality* is that in which the first term is less than the second.

87. *A Compound Ratio* is a ratio formed by the product of all the antecedents of several ratios, for a new antecedent; and of all the consequents, for a new consequent.

88. *A Duplicate Ratio* is the ratio of the squares, *i. e.* of the second powers of the terms.

89. *A Triplicate Ratio* is the ratio of the cubes, *i. e.* of the third powers of the terms.

90. *A Subduplicate Ratio* is the ratio of the square roots of the terms: *e. g.* $\frac{2}{2}, \frac{5}{4}, \frac{3}{5}, \frac{2\cdot3\cdot5}{3\cdot4\cdot6}, \frac{a^2}{b^2}, \frac{a^3}{b^3}, \frac{\sqrt{a}}{\sqrt{b}}$, are respectively examples of these various kinds of ratios.

91. *Proportion* is the equality of ratios, and consists of four terms; the first and third of which are called the antecedents, and the second and fourth the consequents.

:: is the symbol to connect the ratios.

92. *Continued Proportion* is when the consequent of one of the ratios is the antecedent of the next.

93. *Direct Proportion* is that in which the effect produced is greater or less, as the agents or means are greater or less.

94. *Inverse Proportion* is that in which the time of producing the effect is less or greater, as the agency is greater or less; or, it is that in which an agent or agents are diminished or increased, as other agents involved are increased or diminished.

25. *Interest* is money paid for money lent or invested.

96. *The Principal* is money invested or lent.

97. *The Rate per Cent. per Annum* is the interest of £100 for one year.

98. *The Amount* is the principal added to the Interest.

99. *Simple Interest* is the interest of the Principal only.

100. *Compound Interest* for any one year of the time, is the interest of the previous year's amount. The Compound Interest of a Principal for any given time is the sum of the interest for the several years.

101. *Discount* is an abatement made for money paid before it is due.

102. *Bankers' Discount* is simple interest for the given time, at the given rate per cent.

103. *True Discount* is that sum which would amount to the simple interest in the given time, at the given rate per cent.

104. *Present Worth* is the value of a bill or debt after deducting the discount.

105. *Days of Grace* are days over and above the actual date when the bill or debt becomes due, before the expiration of which its recovery is not legal.

N.B. In England *three* days are allowed.

106. *Insurance*, or *Assurance*, is money paid for securing a sum of money, or property, against all contingencies of accident, fire, or loss of life, &c.

107. *Commission*, *Agency*, *Brokerage*, are money paid for services rendered.

108. *The Insurer* or *Underwriter* is the party who undertakes the risk.

109. *The Premium* is the sum paid for the insurance.

110. *The Policy* is the contract for the insurance.

111. *The Gross Weight* is the entire weight before any deductions are made.

112. *The Net Weight* is the weight remaining after all the deductions are made.

113. *The Net Value* is the value after all deductions are made.

114. *Draft* is an allowance to wholesale dealers for the turn of the scale in weighing heavy packages; and to retail dealers, for loss sustained in weighing small parcels of goods.

115. *Tare* is an allowance made for the weight of the package in which the goods are contained, and is generally calculated at so many lbs. per cwt. or per package.

116. *Tret* is an allowance made by wholesale dealers to retail dealers for goods that waste or dry up in keeping. It is usually 4 lbs. in 104 lbs. *i. e.* 1-26th part of the whole, but varies with the nature of the goods.

117. *Stocks* is the name given to the property or capital of the various commercial companies.

118. *Funds*, or *National Debt*, is money lent on Government securities.

119. *A Mortgage* is money advanced on property made over as security for the repayment of the loan.

120. *Debentures* is a term more particularly applied to the mortgages of railway property.

121. "*At Par*" is the term used to denote that coins and precious metals are *at* their standard price; and also that *Stocks*, *Funds*, *Shares*, &c. are *at* their original values.

122. "*At Premium*" is a term used to denote that *Stocks*, *Funds*, &c. are *above* their original values.

123. "*At Discount*" is a term used to denote that *Stocks*, *Funds*, &c. are *below* their original values.

124. *Equation of Payments* denotes that debts, coming due at different dates, are all to be discharged in one payment at such a time that neither debtor nor creditor shall suffer in point of interest.

125. *Exchange* or *Barter* is an equivalent interchange of money or goods, or both.

126. *The Course of Exchange* is the term used by capitalists to denote the equivalents of the standard coins of different countries.

127. *Arbitration of Exchange* is the method of adjusting the exchange between the coins and bills of three or more countries.

128. *The Par of Exchange* is a term used to denote that the *Mint* price of standard bullion in England, and of gold in one or more foreign countries, is the same, so that a true exchange can be effected.

129. *An Arithmetical Progression* is a series of numbers, increasing or decreasing in order, so that each one differs from that preceding it by the same fixed number.

The Common Difference is the difference between any two successive terms.

130. *A Geometrical Progression* is a series of numbers, each one of which is the same fixed multiple, part, or parts of the one preceding it.

The Common Ratio is the multiple, part, or parts that any number is of the one preceding it.

131. *A Series continued ad infinitum* is one which never ends, and whose *exact* value cannot be determined.

132. *The limit of an infinite Series* is that approximate value of it, which differs from the actual value by a quantity so small as not to be appreciable.

133. *Variations* or *Permutations* are the different arrangements of a number of quantities as to their relative position.

134. *Combinations* are the number of sets of twos, threes, &c., that can be formed out of a number of quantities, without reference to their relative position.

CHAPTER II.

RULES AND THEIR PROOFS, WITH REMARKS THEREON.

1. *Simple Rules* are those which involve abstract quantities, or only concrete quantities of one kind, and of one and the same denomination.

2. *Compound Rules* involve concrete numbers with their different denominations.

3. *The Rule of the Radix* is as follows:—

The radix of the ordinary scale is 10, and defines the second denomination;

(The radix)^2 of the ordinary scale is 100, and defines the third denomination;

(The radix)^3 of the ordinary scale is 1000, and defines the fourth denomination; and so on.

Thus 5 represents five units.

45 represents four times the radix, together with five units.

345 represents three times the (radix)^2, together with four times the (radix), and five units; and so on.

If the *radix* were 4, instead of 10, then, in the number 1232—

The first figure 2 represents the units.

The second figure 3 represents 3 × 4, or three times the radix.

The third figure 2 represents $2 \times (4)^2$, or twice the $(\text{radix})^2$.

The fourth figure 1 represents $1 \times (4)^3$, or once the $(\text{radix})^3$; and so for any radix whatever.

The first figure on the right hand in any number is *specially* the units' figure.

The second figure is the radix figure, or so many times the radix.

The third figure is the radix figure of the radix, or so many times the $(\text{radix})^2$; and so on.

No figure in any number can be so great as the radix; consequently, in casting up a column of units, if the amount be equal to or greater than the radix, and less than the $(\text{radix})^2$, it will consist of two figures—the first to the right representing the units, and the second the number of times the radix is contained. If it be equal to or greater than the $(\text{radix})^2$, and less than the $(\text{radix})^3$, it will consist of three figures—the third representing the number of times the $(\text{radix})^2$ is contained; and so on.

Any figure placed before another becomes a radix figure to the first: thus, if 5 represent five units, then 45 represents forty-five units; if 5 represent five tens, then 45 represents forty-five tens; and so on.

4. *Addition.—Rule.* Place the numbers to be added one under the other, so that the units form one column, the tens another, and so on. Cast up the first or units' column, put down the units of the sum under this column, and carry the radix or tens' figure to the tens' column. Then cast up the second or radix column, adding in the radix figure carried, and treat the amount as before, only putting what would have been the units' figure under the tens' or radix column, carry the $(\text{radix})^2$ or hundreds' figure to the hundreds' column, and the $(\text{radix})^3$, or thousands' figure, if any, to the thousands' column, and proceed as before till the addition is complete.

N.B. As a unit is one of *any* kind, if a number be said to contain forty-five tens, or be represented in figures by 45 tens, we call the 5, units of tens, and the 4, the *radix figure of the radix* figure, or *tens of tens*; and so of all the other denominations.

5. *Subtraction.—Rule.* Place the *Subtrahend* under the *Minuend*, the different denominations being respectively under one another.

If all the figures of the minuend be greater than the respective figures of the subtrahend, their differences can be at once detected and put down in order.

If the units' figure of the minuend be the less, add to it one from the tens of the minuend, and subtract, putting down the difference (which must be units) under the units. If the number of tens thus diminished be not less than the number in the subtrahend, the difference can be found; but if it be less, then add one from the hundreds to it, and subtract. The remainder must be tens, which put down under the tens. If the number of hundreds thus diminished be less than the number in the subtrahend, add one from the thousands to it, and proceed as before until the subtraction is complete.

$$\begin{array}{lrl} \text{Ex.} & 34567 & \text{minuend.} \\ & \underline{25478} & \text{subtrahend.} \\ & \underline{9089} & \end{array}$$

Here, 8 from 7 we cannot take; add 1 from the 6 tens to the 7 units, and the sum will be 17 units; then 8 from 17 will leave 9.

Again, 7 tens from the 5 tens remaining we cannot take; add one from the 5 hundreds to the 5 tens, and the sum will be 15 tens; then 7 from 15 will leave 8.

4 hundreds from the 4 hundreds remaining will leave 0.

5 thousands from 4 thousands we cannot take; add one from the 10 thousands to the 4 thousands, and the sum will be 14 thousands; then 5 from 14 will leave 9.

Lastly, 2 ten-thousands from the 2 ten-thousands remaining will leave 0.

6. Units, tens, hundreds, &c. multiplied by units, give units, tens, hundreds, &c. for a result. Units,

tens, hundreds, &c. multiplied by tens, give tens, hundreds, thousands, &c. respectively; and so on.

Only abstract numbers can be multipliers, for we cannot multiply £3 by £4, and say the result is £12; *i.e.* we could not repeat £3, £4 times.

7. *Multiplication.—Rule.* Place the multiplier under the multiplicand, units under units, tens under tens, &c. First, multiply each figure in the multiplicand by each figure in the multiplier separately, commencing with the units; put down the units' figure of the product, and carry the radix figure, which will be tens. Next, multiply the tens, add in the number carried, and put down the units' figure of the result, which will be tens, in the tens' column, and carry the radix figure, which will be hundreds; and so on, till each figure of the multiplicand has been multiplied by the units of the multiplier.

Again, proceed to do the same with the tens of the multiplier, recollecting that the units' figure of the first result will be tens, and therefore must be placed in the tens' column.

Proceed in the same way with the remaining figures of the multiplier.

We thus obtain a series of columns of units, tens, hundreds, &c. which we cast up as in addition.

It is unimportant by which figure we multiply first, if attention be paid to Art. 6, and the results grouped under their respective columns.

E.g. Let 1234 be multiplied by 567. Multiply first by the tens, then by the units, and lastly by the hundreds, and the result stands as below :—

$$\begin{array}{r} 1234^{(1)} \\ 567 \\ \hline 7404 \\ 8638 \\ 6170 \\ \hline 699678 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 1234^{(2)} \\ 567 \\ \hline 8638 \\ 7404 \\ 6170 \\ \hline 699678 \\ \hline \end{array}$$

We shall proceed to discuss (2), as being the ordinary mode.

7 times 4 units are 28 units ; put down the 8 units in the units' column, and carry the radix figure 2, which represents tens.

7 times 3 tens are 21 tens, and the 2 tens carried make 23 tens; put down the 3 tens under the tens, and carry the radix figure 2, which represents hundreds.

7 times 2 hundreds are 14 hundreds, which, with the 2 hundreds carried, make 16; put down the 6 under the hundreds, and carry the 1, which is 1 thousand.

7 times 1 thousand are 7 thousands, which, with the 1 thousand carried, make 8; put it down under the thousands.

Proceed in the same way with the remaining figures of the multiplier.

8. In *Division* it is necessary to attend to the property of the radix. (Art. 3.)

Any *number* standing before a figure, or number, becomes its *radix number; i. e.* adds to it so many times *ten* of that denomination which it happens to be.

Thus, in 345, 34 is a radix number to 5, and adds to the *five units* thirty-four tens; *i. e.* three hundred and forty.

In 12345, the number 123, standing before 45, adds to it *one hundred and twenty-three hundreds.*

Conversely, in any *number*, if we consider only a certain number of the figures composing it, beginning with the highest denomination, they will represent so many of the denomination denoted by the last of these figures.

Thus, in the number 12345, the figures 1234 represent *tens*, because the last figure 4 is tens.

The figures 123 represent hundreds, because the last figure 3 is hundreds; and so on.

The number may be expressed thus:—

10000 = one ten thousand
2000 = two thousands
300 = three hundreds
40 = four tens
5 = five units.
12345

If we consider the first and second lines only, we get 12000; *i. e.* twelve *thousands.* If we take in the third line, we get 12300; *i. e.* one hundred and twenty-three *hundreds.* If the fourth also, we get 12340; *i. e.*

one thousand two hundred and thirty-four *tens*. If the fifth likewise, we get 12345; *i. e.* twelve thousand three hundred and forty-five *units*.

9. *Division.* If the dividend be abstract, the divisor must be abstract, and the quotient will be abstract; but if the dividend be concrete, the divisor may be abstract or concrete, and then the quotient will be concrete or abstract respectively.

Thus, $20 \div 4 = 5$ abstract
$£20 \div 4 = £5$ concrete
$£20 \div £4 = 5$ abstract.

Rule.

(i.) *Short Division*, or that in which the divisor is any one of the digits: sometimes 10, 11, 12, are included among the divisors.

Divide the first figure of the dividend, or that representing the highest denomination, by the divisor, if possible; if not, take in the next figure, to which the first will be a radix figure, and the quotient will be of the same denomination as the last figure taken, and must be placed under that figure. The remainder, if any, becomes a radix figure to the next figure, and divide as before. The quotient will be of this next denomination, and must be put down accordingly. The remainder, if any, will be the radix figure to the next figure; then proceed as before till the division is complete. The final remainder, if any, is the numerator of a fraction of which the divisor is the denominator.

(ii.) *Long Division; i. e.* in which the divisor consists of more than one figure.

Place the divisor and dividend in one line; thus,

divisor) dividend (quotient to be found.

Take as many figures of the dividend, beginning with the first to the left, as make a number equal to, or next greater than, the divisor for a first dividend. By Art. 8, it will be of that denomination represented by the last figure. Divide, and the quotient will be of the same denomination. Find the product of the quotient and divisor, and subtract it from the first dividend. The remainder, if any, will be the *radix number* of the next figure of the original dividend, and, with it, forms the next partial dividend. Divide, and proceed as before.

Example:—Divide 167790 by 329

329) $16\dot{7}790$ (510
1645
329
329
0

CHAPTER III.

TABLES.

10. MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

11. MONEY TABLE.

4 Farthings make 1 Penny, *d.*
12 Pence „ 1 Shilling, *s.*
20 Shillings „ 1 Pound or Sovereign, £.

£ *s.* *d.* are respectively the initial letters of the

Latin words, Libra (pound), solidus (shilling), denarius (penny).

Obsolete Coins.—A Guinea, 21*s.* A Moidore, 27*s.* A Mark, 13*s.* 4*d.* A Noble, 6*s.* 8*d.*

Other Coins in use.—Groat, 4*d.* Florin, 2*s.* Crown, 5*s.*

1 ℔ of gold is coined into 46$\frac{29}{40}$ sovereigns.

1 ℔ Troy of silver is coined into 66 shillings.

The *Mint* value of standard gold is £3 17*s.* 10½*d.* per ounce Troy; and of standard silver, 5*s.* 6*d.*

24 pence are made from an Avoirdupois pound of copper.

EXTENDED MONEY TABLE.

FARTHINGS.			PENCE.			PENCE.				SHILLINGS.			
qrs.		*d.*	*d.*		*s.*	*d.*		*s.*	*d.*	*s.*		£	*s.*
4	are	1	12	are	1	20	are	1	8	20	are	1	0
6	,,	1½	24	,,	2	30	,,	2	6	30	,,	1	10
8	,,	2	36	,,	3	40	,,	3	4	40	,,	2	0
10	,,	2½	48	,,	4	50	,,	4	2	50	,,	2	10
12	,,	3	60	,,	5	60	,,	5	0	60	,,	3	0
14	,,	3½	72	,,	6	70	,,	5	10	70	,,	3	10
16	,,	4	84	,,	7	80	,,	6	8	80	,,	4	0
18	,,	4½	96	,,	8	90	,,	7	6	90	,,	4	10
20	,,	5	108	,,	9	100	,,	8	4	100	,,	5	0
22	,,	5½	120	,,	10	110	,,	9	2	110	,,	5	10
24	,,	6	132	,,	11	120	,,	10	0	120	,,	6	0
26	,,	6½	144	,,	12	130	,,	10	10	130	,,	6	10
28	,,	7	156	,,	13	140	,,	11	8	140	,	7	0
30	,,	7½	168	,,	14	150	,,	12	6	150	,,	7	19
32	,,	8	180	,,	15	160	,,	13	4	160	,,	8	0
34	,,	8½	192	,,	16	170	,,	14	2	170	,,	8	10
36	,,	9	204	,,	17	180	,,	15	0	180	,,	9	0
38	,,	9½	216	,,	18	190	,,	15	10	190	,,	9	10
40	,,	10	228	,,	19	200	,,	16	8	200	,,	10	0
42	,,	10½	240	,,	20	210	,,	17	6	210	,,	10	10
44	,,	11				220	,,	18	4	220	,,	11	0
46	,,	11½				230	,,	19	2	230	,,	11	10
48	,,	12 or 1*s.*				240	,,	20	0	240	,,	12	0

12. ALIQUOT PARTS.

OF A £.

s.	*d.*		£
10	0	are	$\frac{1}{2}$
6	8	"	$\frac{1}{3}$
5	0	"	$\frac{1}{4}$
4	0	"	$\frac{1}{5}$
3	4	"	$\frac{1}{6}$
2	6	"	$\frac{1}{8}$
2	0	"	$\frac{1}{10}$
1	8	"	$\frac{1}{12}$
1	4	"	$\frac{1}{15}$
1	3	"	$\frac{1}{16}$
1	0	"	$\frac{1}{20}$
	10	"	$\frac{1}{24}$
	8	"	$\frac{1}{30}$
	6	"	$\frac{1}{40}$
	5	"	$\frac{1}{48}$
	4	"	$\frac{1}{60}$
	3	"	$\frac{1}{80}$
	2	"	$\frac{1}{120}$
	1	"	$\frac{1}{240}$

OF A SHILLING.

d.		*s.*
6	are	$\frac{1}{2}$
4	"	$\frac{1}{3}$
3	"	$\frac{1}{4}$
2	"	$\frac{1}{6}$
$1\frac{1}{2}$	"	$\frac{1}{8}$
1	"	$\frac{1}{12}$

OF A PENNY.

qrs.		*d.*
2	are	$\frac{1}{2}$
1	"	$\frac{1}{4}$

OF A TON.

cwt.		*ton.*
10	are	$\frac{1}{2}$
5	"	$\frac{1}{4}$
4	"	$\frac{1}{5}$
2 3 *qrs.* 12 *lbs.*		$\frac{1}{7}$
$2\frac{1}{2}$	"	$\frac{1}{8}$
2	"	$\frac{1}{10}$
1	"	$\frac{1}{20}$

OF A CWT.

qrs. *lbs.*		*cwt.*
2, or 56	are	$\frac{1}{2}$
1, or 28	"	$\frac{1}{4}$
16	"	$\frac{1}{7}$
14	"	$\frac{1}{8}$
8	"	$\frac{1}{14}$
7	"	$\frac{1}{16}$
4	"	$\frac{1}{28}$
2	"	$\frac{1}{56}$
1	"	$\frac{1}{112}$

OF A QUARTER.

lbs.		*qr.*
14	are	$\frac{1}{2}$
7	"	$\frac{1}{4}$
4	"	$\frac{1}{7}$
$3\frac{1}{2}$	"	$\frac{1}{8}$
2	"	$\frac{1}{14}$
$1\frac{3}{4}$	"	$\frac{1}{16}$
1	"	$\frac{1}{28}$

OF A LB. AVOIRDUPOIS.

oz.		*lb.*
8	are	$\frac{1}{2}$
4	"	$\frac{1}{4}$
2	"	$\frac{1}{8}$
1	"	$\frac{1}{16}$

OF A LB. TROY.

oz.		*lb.*
6	are	$\frac{1}{2}$
4	"	$\frac{1}{3}$
3	"	$\frac{1}{4}$
2	"	$\frac{1}{6}$
$1\frac{1}{2}$	"	$\frac{1}{8}$
1	"	$\frac{1}{12}$

OF AN OZ. TROY.

dwts.		*oz.*
10	are	$\frac{1}{2}$
5	"	$\frac{1}{4}$
4	"	$\frac{1}{5}$
$2\frac{1}{2}$	"	$\frac{1}{8}$
2	"	$\frac{1}{10}$
1	"	$\frac{1}{20}$

OF A DWT.

grs.		*dwt.*
12	are	$\frac{1}{2}$
8	"	$\frac{1}{3}$
6	"	$\frac{1}{4}$
4	"	$\frac{1}{6}$
3	"	$\frac{1}{8}$
2	"	$\frac{1}{12}$
1	"	$\frac{1}{24}$

13. TABLE OF TROY WEIGHT.

24 Grains (*grs.*)	make	1 Pennyweight, *dwt.*
20 Pennyweights	„	1 Ounce, *oz.*
12 Ounces	„	1 Pound, *lb.*

This table derives its name from *Troyes*, a town in the Province of Champagne, now in the Department of the Aube, in France, whence it was introduced into this country by William the Conqueror.

14. TABLE OF APOTHECARIES' WEIGHT.

20 Grains (Troy)	make	1 Scruple, *sc.*
3 Scruples	„	1 Dram, *dr.*
8 Drams	„	1 Ounce, *oz.*
12 Ounces	„	1 Pound, *lb.*

15. TABLE OF AVOIRDUPOIS WEIGHT.

16 Drams	make	1 Ounce, *oz.*
16 Ounces	„	1 Pound, *lb.*
28 Pounds	„	1 Quarter, *qr.*
4 Quarters, or 112 lbs.		1 Hundredweight, *cwt.*
20 Hundredweight, or 2240 lbs	„	1 Ton, *T.*

1 Stone = 14 lbs.

Avoir du poids, or *Avoirdupois Weight*, which was a medium between the French and the ancient English weights, arose in consequence of the dissatisfaction of the English with Troy weight, because the pound of the latter did not weigh so much as the pound in use at that time in this country. Avoirdupois Weight was first made legal in the reign of Henry VII., and it is used in weighing provisions and heavy articles generally.

16. *Comparison of Weights.*

The origin of all *Weights and Measures* of solids in England was derived from a grain of wheat, and statutes were passed in the reigns of Henry III., Edward I., and Henry VII., which enacted that 32 of these grains, well dried and gathered from the middle of the ear, were to make 1 pennyweight, from being equal in weight to the silver penny then in use.

It was subsequently thought better to divide the pennyweight into 24 equal parts, called grains.

1 Ounce Avoirdupois is 437½ Grains.
1 Ounce Troy „ 480 „
1 Pound Troy „ 5760 „
1 Pound Avoirdupois „ 7000 „
175 lbs. Troy = 144 lbs. Avoirdupois.
175 oz. Troy = 192 oz. Avoirdupois.

17. *Linear or Long Measure.*

12 Inches make 1 Foot, *ft.*
3 Feet, or 36 Inches „ 1 Yard, *yd.*
5½ Yards, or 16½ Feet „ 1 Rod, Pole, or Perch, *po.*
40 Poles, or 10 Chains, or 220 Yards „ 1 Furlong, *fur.*
8 Furlongs, or 1760 Yards „ 1 Mile, *m.*
3 Miles „ 1 League, *l.*

Besides these, are: Line = one-twelfth of an inch, Hand = 4 inches (used in measuring the height of horses), Span = 9 inches, Cubit = 18 inches, Military Pace = $2\frac{1}{2}$ feet, Geometrical Pace = 5 feet. 60 Geographical Miles, or $69\frac{1}{10}$ (nearly) English Miles, make 1 Degree of the Meridian; Fathom = 2 yards or 6 feet.

The Chain of 4 poles, or 22 yards, consists of 100 Links, each Link being $7\frac{23}{25}$, or 7·92 inches in length.

18. *Cloth Measure.*

$2\frac{1}{4}$ Inches	make	1 Nail, *n.*
4 Nails, or 9 Inches	„	1 Quarter of a Yard, *q.*
4 Quarters	„	1 Yard, *yd.*
5 Quarters	„	1 English Ell, *E. e.*
3 Quarters	„	1 Flemish Ell, *Fl. e.*
6 Quarters	„	1 French Ell, *Fr. e.*

Used for all Drapery goods.

19. *Superficial or Square Measure.*

144 Square Inches (*sq. in.*)	make	1 Sq. Foot, *sq. ft.*
9 Square Feet	„	1 Sq. Yard, *sq. yd.*
$30\frac{1}{4}$ Sq. Yds., or $272\frac{1}{4}$ Sq. Ft.	„	1 Sq. Rod, Pole, or Perch.
40 Perches	„	1 Rood, *r.*
4 Roods, or 4840 Yards	„	1 Acre, *a.*
10,000 Square Links	„	1 Sq. Chain.
10 Sq. Chains, or 100,000 Links	„	1 Acre.
640 Acres	„	1 Sq. Mile.

20. *Cubic or Solid Measure.*

1728 Cubic Inches (*c. in.*)	make	1 Cubic Foot, *c. ft.*
27 Cubic Feet	”	1 Cubic Yard, *c. yd.*
40 Cubic Feet of Round Timber, or 50 Cubic Feet of Hewn Timber	”	1 Ton, or Load.
42 Cubic Feet	”	1 Ton of Shipping.

A Cubic Foot of Water weighs 1000 ozs. Avoirdupois.
A Cord of Wood is 128 Cubic Feet.

21. *Wine and Spirit Measure.*

4 Gills*	make	1 Pint, *pt.*
2 Pints	”	1 Quart, *qt.*
4 Quarts	”	1 Gallon, *gal.*

Measures which vary with the nature of the wines, and also with the custom of the country from which they come :—

63 Gallons	make	1 Hogshead, *hhd.*
84 Gallons	”	1 Puncheon, *pun.*
2 Hhds. or 126 Gals.	”	1 Pipe or Butt.
4 Hhds. or 252 Gals.	”	1 Tun.

---

* A gill is ¼ pint London measure.

22. *Ale, Beer, and Porter Measure.*

2 Pints	make	1 Quart.
4 Quarts	„	1 Gallon.
9 Gallons	„	1 Firkin.
2 Firkins, or 18 Gals.	„	1 Kilderkin.
2 Kilderkins, or 36 Gals.	„	1 Barrel.
3 Kilderkins, or 54 Gals.	„	1 Hogshead.
2 Hogsheads, or 108 Gals.	„	1 Butt.

23. *Corn or Dry Measure.*

2 Pints	make	1 Quart.
4 Quarts	„	1 Gallon.
2 Gallons	„	1 Peck.
4 Pecks	„	1 Bushel.
2 Bushels	„	1 Strike.
8 Bushels	„	1 Quarter.
5 Quarters	„	1 Wey, or Load.
2 Loads	„	1 Last.

24. *Miscellaneous Table.*

12 Articles	make	1 Dozen.
12 Dozens	„	1 Gross.
20 Articles	„	1 Score.
6 Scores	„	1 Long Hundred.
24 Sheets of Paper	„	1 Quire.
20 Quires	„	1 Ream.
43 Quires	„	1 Printer's Bundle.
10 Reams	„	1 Bale.

25.

Table of Time.

60 Seconds (*sec.*)	make	1 Minute, *min.*
60 Minutes	„	1 Hour, *hr.*
24 Hours	„	1 Day, *d.*
7 Days	„	1 Week, *w.*
52 wks. 1 d. 6 hrs., or 365 days, 6 hrs.		1 Julian Year
365 days, 5 hrs. 48 m. $47\frac{1}{2}$ sec.		1 Solar Year.
100 Years		1 Century.

The Quarterly Terms.

Lady-Day,	March 25th.
Midsummer,	June 24th.
Michaelmas,	September 29th.
Christmas,	December 25th.

A Year is divided into 12 Calendar Months, the number of days in each of which is easily remembered by means of the following lines:—

Thirty days hath rich September,
April, June, and dark November;
February has twenty-eight alone,
And all the rest have thirty-one:
But Leap-year coming once in four,
February then has one day more.

26. *Table of Angular Measure.*

60 Seconds (″)	make	1 Minute, ′.
60 Minutes	„	1 Degree, °.
90 Degrees	„	1 Quadrant.
360 Degrees, or 4 Quadrants	„	1 Circle.

27. *Compound Addition.—Rule.* Arrange the items to be added in such a manner that the different denominations shall form separate columns. Cast up the column of the lowest denomination first, find how many of the next denomination are contained in the sum, put down the remainder, if any, and cast up the number of the next denomination with the next column. Proceed in the same manner with all the columns: the last, or highest denomination, will be dealt with as in Simple Addition.

28. *Compound Subtraction.—Rule.* Place the subtrahend under the minuend, and subtract the number of the lowest denomination of the latter from that of the former, and put down the remainder, if any. If the number in the subtrahend be the greater, borrow one from the next higher denomination of the minuend, and add it, when reduced, to the number of the lower denomination of the minuend; then subtract as before.

From the number of the next denomination of the minuend less one, subtract, if possible, that of the subtrahend; if not, borrow one as before from the next higher denomination of the minuend, add it when reduced, and subtract as above.

Proceed thus till all the denominations have been dealt with.

29. *Compound Multiplication.—Rule.* Consider each denomination of the multiplicand as a separate multiplicand, and find the product of the lowest and the multiplier. See how many of the next higher denomination are contained in it, put down the remainder, if any, and carry the number of the higher to the product formed by multiplying the next denomination of the multiplicand by the multiplier. Treat this in the same way as the first, and proceed in like manner till the whole multiplicand has been multiplied.

To multiply by a large number.

(i.) Find factors (not greater than any of the nine digits) which compose the number. Multiply by one of them, this product by the next, and the second product by the third; and so on.

(ii.) If the number consist of two, three, four, five, &c. figures, multiply by 10 in the first case, that product by 10 again for the second case, that product by 10 again for the third case; and so on. Multiply the last of these products by the figure of the multiplier denoting the highest denomination. To this last product add the product of the multiplicand by the units' figure, the product of the tens' line by the number of tens, the hundreds' line by the number of hundreds; and so on for all except the last. These items, when added, will give the entire product required.

Ex. gr. Multiply £2 14*s.* 6½*d.* by 23456.

	£	*s.*	*d.*	
6 ×	2	14	6½	
			10	
5 ×	27	5	5	= 10 times, or tens' line.
			10	
4 ×	272	14	2	= 100 times, or hundreds' line.
			10	
3 ×	2727	1	8	= 1000 times, or thousands' line.
			10	
	27270	16	8	= 10000 times, or ten thousands' ,,
			2	
	54541	13	4	= 20000 times.
	16	7	3	= 6 ,,
	136	7	1	= 50 ,,
	1090	16	8	= 400 ,,
	8181	5	0	= 3000 ,,
	63966	9	4	= 23456 ,,

30. Concrete numbers of one kind and of different denominations can be compared and divided by concrete numbers of the same kind, by bringing them to one and the same denomination. Thus, 4*s.* 1*d.* may be divided by 1s. 0$\frac{1}{4}$*d.* by reducing them both to farthings.

4*s.* 1*d.* = 196 farthings.
1*s.* 0$\frac{1}{4}$*d.* = 49 farthings.
196 divided by 49 = 4.

Division.—Rule. Divide the part of the dividend representing the highest denomination by the divisor, as in Simple Division, and the quotient will be of that denomination.

Reduce the remainder, if any, to the next lower denomination, adding in the number representing the same denomination in the dividend for a new dividend. Divide as before, and the quotient will be of that denomination. Proceed in like manner till all the denominations of the dividend have been taken in.

N.B.—The divisor must be abstract.

CHAPTER IV.

VULGAR AND DECIMAL FRACTIONS.

1. As every whole abstract or concrete number of one kind is made up of units, it may be written as a fraction by merely putting the unit, of which it is composed, under it as a denominator. Thus, $4 = \frac{4}{1}$.

2. So any number may be expressed as a fraction of any other, by writing the latter under the former for a denominator.

Ex. What fraction is 4 of 5?
1 is one-fifth of 5 = $\frac{1}{5}$
2 are two-fifths of 5 = $\frac{2}{5}$
3 are three-fifths of 5 = $\frac{3}{5}$
Therefore, 4 are four-fifths of 5 = $\frac{4}{5}$

3. Also any concrete number of different denominations may be expressed as a fraction of any other concrete number of the same kind, by reducing them both to the lowest denomination mentioned in either, and proceeding as in the last article.

4. When we speak of any fraction,—as, for instance, two-thirds ($\frac{2}{3}$),—we mean (1) that some whole number

or quantity is divided into *three* equal parts, of which we take *two* only; or (2) that two numbers or quantities are taken, of which *one-third* is the *unit.* So in the fraction three-fourths ($\frac{3}{4}$), we mean that a whole number or quantity is divided into *four* equal parts, of which *three* only are taken; or that *three* quantities or numbers are taken, of which one-fourth is the unit.

5. If we wish to represent the whole number 4 as a fraction whose unit is one-third, *i. e.* with a denominator 3, we proceed thus:—

1 whole contains *three* such units
2 wholes contain *six* ”
3 ” *nine* ”
4 ” *twelve* ”

and ∴ the fraction required is $\frac{12}{3}$.

Thus we can represent any whole number as an improper fraction with any denominator we please, by multiplying it by that denominator for a numerator.

6. To compare fractions, *i. e.* to find the greatest or the least, or to find the excess of one above the other, or their sum, they must have a common denominator. We cannot add two-thirds to three-fourths, and say the sum is five-thirds or five-fourths, any more than we can add two shillings to threepence, and say there are five shillings or five pence.

7. To find the *Least Common Denominator,* or least common multiple of the denominators, place all the denominators in one line.

(i.) Cross out any that are measures of one or more of the others, because a multiple of any number must be a multiple of its measures.

(ii.) Take out the greatest for a divisor, and cross out all the factors in the others common to it, for the same reason as before, putting down in another line those which have no common factor, and the remaining factors of the rest.

(iii.) Take out again the greatest of these, and proceed in the same manner till the numbers left have no common factor.

(iv.) Multiply all the divisors and remaining factors together for the Least Common Multiple, which will also be the Least Common Denominator.

Ex. (i.) To find the least common multiple of the numbers 3, 4, 5, 9, 18, 40, 55, 27.

Here 3 measures 9, and 9 measures 18;
4 measures 40, and 5 measures 55;
∴ 3, 9, 4, 5 may be struck out.

$$\begin{array}{r|l}
 & \cancel{3},\ \cancel{4},\ \cancel{5},\ \cancel{9},\ 18,\ 40,\ \cancel{55},\ 27 \\ \hline
55 & 18,\ 40,\ \cancel{27} \\ \hline
27 & 18,\ \cancel{8}. \\ \hline
8 & \cancel{2} \\ \hline
\end{array}$$

The greatest, 55, is taken out for a divisor;

of 18, 40, 27, and 55, 55 and 40 have a common measure, 5; but 55 and 18, and 55 and 27, have no common measure; 40 is divided by 5, and the quotient, 8, together with 18, is put down; 27 being the greatest is taken out for a divisor; 27 and 18 have a common measure, 9; but 27 and-8 have no common measure. 18 is divided by 9, and the quotient 2 is put down; 8 being the greatest is taken out for a divisor; 2 being a measure of 8 is struck out.

Therefore, L. C. M. $=55\times27\times8=11880$.

Ex. (ii.) Find L. C. M. of 5, 7, 12, 14, 16, 18, 20, 35, 40, 50.

$$\begin{array}{l}
\cancel{5},\ \cancel{7},\ 12,\ 14,\ 16,\ 18,\ \cancel{20},\ 35,\ 40,\ \cancel{50} \\ \hline
50/12,\ 14,\ 16,\ 18,\ 35,\ 40 \\ \hline
9/6,\ 7,\ 8,\ \cancel{9},\ 7,\ 4 \\ \hline
8/2,\ 7,\ \cancel{8},\ 7,\ 4 \\ \hline
7/\cancel{7},\ \cancel{7} \\ \hline
\end{array}$$

Therefore, L. C. M. $=50\times9\times8\times7=25200$.

8. To find the *Greatest Common Measure* of two numbers.

Divide the greater by the less.

If the remainder be a measure of the *divisor*, it is also a measure of the *dividend*.

It is likewise the greatest factor common to them, and therefore the greatest common measure of the two numbers.

If the remainder be not a measure of the *divisor*,

Make the remainder a divisor, and the last divisor a dividend.

If the remainder in this case be a measure of the divisor, it is also a measure of the dividend, and consequently a measure of the first divisor, and therefore, as before, the greatest common measure of the two numbers; but if not,

Proceed in the same way until there is no remainder.

The last divisor will be the G. C. M. required.

9. To bring fractions to a common denominator.

(i.) Multiply each numerator by all the denominators except its own for a series of new numerators, and all the denominators together for a new denominator.

(ii.) Or, find the least common multiple of the denominators, as in the last Art., which will be the least common denominator. Divide it by each denominator, and multiply the numerator by the quotient for a new numerator.

10. To multiply a fraction by a whole number, multiply the numerator by the whole number.

Since the denominator merely points out the particular kind of parts into which the whole has been divided, it is clear that if we double the numerator, we shall have twice as many of those parts; if we treble it, we shall have three times as many; and so on.

This will be still more clearly seen if, instead of writing the fraction in the ordinary way (as $\frac{2}{3}$ for instance), we express it by a new symbol, thus 2_3; just as we write 2*s*. to express two shillings. If then we multiply 2*s*. by 4, we get a product 8*s*., a number in which the unit (a shilling) remains the same. So in the same manner 2_3 multiplied by 4 gives a product 8_3, or, using the old mode of expressing it, $\frac{8}{3}$, the unit remaining the same.

11. To multiply a fraction by a fraction.

Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

For since $\frac{1}{2}$ multiplied by 1 gives a product $\frac{1}{2}$,
Then $\frac{1}{2}$,, $\frac{1}{2}$,, only half as large, or $\frac{1}{4}$.

If now a line be divided into *four* equal parts, one-half of it contains *two* of these parts, and ∴ the half of one-half contains only *one* of them, *i. e.* one-fourth of the whole; ∴ halves multiplied by halves give fourths.

Again, $\frac{1}{2}$ multiplied by $\frac{1}{3}$ gives a product only one-third as large.

If a line be divided into six equal parts, then one-half of it contains *three* of these parts, and one-third of one-half contains only *one* of these parts, *i. e.* one-sixth of the whole.

∴ halves multiplied by thirds give sixths.

Again, $\frac{1}{5}$ multiplied by 1 gives a product $\frac{1}{5}$
$\frac{1}{2}$,, $\frac{1}{7}$,, only one-seventh as large.

If a line be divided into *thirty-five* equal parts, one-fifth of it will contain *seven* of these parts, and one-seventh of one-fifth will contain only *one* of these parts, *i. e.* one thirty-fifth of the whole.

∴ fifths multiplied by sevenths give thirty-fifths.

Suppose, then, we wish to multiply $\frac{3}{4}$ by $\frac{2}{3}$, the product will be *twelfths.*

Now, $\frac{1}{4}$ multiplied by $\frac{1}{3}$ is $\frac{1}{12}$
$\frac{1}{4}$,, $\frac{2}{3}$ is $\frac{2}{12}$
∴ $\frac{3}{4}$,, $\frac{2}{3}$ will be three times as great a product, *i. e.* $\frac{6}{12}$.

In this manner the truth of the rule may be established.

12. To divide a fraction by a whole number.

(i.) Divide the numerator by it, if possible, because the denominator only points out the kind of parts, and has nothing to do with the number taken. Thus $\frac{8}{9}$ denotes 8 ninths, which divided by 4 gives a quotient of 2 ninths, or $\frac{2}{9}$, and so generally.

(ii.) Multiply the denominator by it.

If the number of the equal parts into which any whole has been divided be doubled, trebled, &c., then each of these new parts will be half the size, a third of the size, &c. respectively of the original parts. Therefore, if the numerator remain the same, the fraction will have been divided by 2, 3, &c. respectively.

13. To divide a fraction by a fraction. Invert the divisor, and multiply them together.

Any number	divided	by	1 gives the same number for a quotient		
"	"		$\frac{1}{2}$ gives a quotient twice as large.		
"	"		$\frac{1}{3}$	"	thrice "
			and so on.		
∴ if $\frac{2}{3}$	be divided	by	$\frac{1}{2}$	the quotient is	$\frac{4}{3}$
"	"		$\frac{1}{4}$	"	$\frac{8}{3}$
Also if $\frac{2}{5}$	"		$\frac{1}{2}$	"	$\frac{4}{5}$
"	"		$\frac{1}{3}$	"	$\frac{6}{5}$
"	"		$\frac{2}{3}$	"	only half as large, *i. e.* $\frac{3}{5}$ or $\frac{6}{10}$, Art. 11.
"	"		$\frac{1}{7}$	"	$\frac{14}{5}$
"	"		$\frac{3}{7}$	"	only one-third as large, *i. e.* $\frac{14}{15}$, Art. 11.

Thus the rule may be established generally.

14. If the numerator and denominator of a fraction be both multiplied or both divided by the same number, the fraction is not altered in value. Any number multiplied or divided by 1 will remain the same; and $1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4}$, &c. Consequently, to multiply or divide the numerator and denominator of a fraction by the same number, is the same as multiplying it by the fraction $\frac{2}{2}$ or $\frac{3}{3}$, &c.; *i. e.* by 1. Its value is therefore unaltered.

15. The continued product of any number of fractions is obtained by multiplying all the numerators

together for a new numerator, and all the denominators together for a new denominator. This will give the resulting fraction in its highest terms.

16. We have seen that if the numerator and denominator of a fraction be both divided by the same number, its value is not changed. Therefore, factors common to any numerator and denominator may be struck out. This is called cancelling; and the object is, to present the resulting fraction (of a continued product of fractions) in its lowest terms.

17. The following is a continuation of the Numeration Table in Chapter I. (12):—

Thousands.	Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred thousandths.	Millionths.	Ten Millionths.	&c. &c.
4	3	2	1	·	1	2	3	4	5	6	7	

i. e. the decimal radix is $\frac{1}{10}$.

18. To convert Decimal Fractions into Vulgar Fractions.

Express the denominator.

In Decimal Fractions the denominator will be denoted by the denomination of the last figure to the right of the decimal part. Thus, in 2345·678, the 8 is 8 thousandths, therefore the denominator is 1000. Again, in 23·4, the 4 is 4 tenths, therefore the denominator is 10.

19. From this we see that, in division, the remainder from the units becomes the radix figure of the tenths, and the remainder from the tenths is the radix figure of the hundredths; and so on. Therefore, if, in dividing any number by another, we wish to proceed with the division after the units, we must suppose cyphers annexed to the dividend, but separated by the decimal point from the original dividend to mark where the radix changes from ten to one-tenth.

Ex. gr. 23 = 23·00.

Therefore, dividing by 4, we get—

$$\begin{array}{r} 4\,/\,\underline{23{\cdot}00} \\ \underline{5{\cdot}75} \end{array}$$

Because 4 is not contained in 2, therefore the 2 becomes the radix figure of 3; and 4 is contained five times in twenty-three, with a remainder 3. Now 3 becomes the radix figure of the next denomination, which is tenths, and makes 30 tenths. 4 divides 30 tenths, and gives 7 tenths for a quotient, with remainder 2. This becomes the radix figure of the next denomination, which is hundredths. Again: 4 divides 20 hundredths, and gives a quotient of 5 hundredths.

20. *Reduction of Decimal Fractions.*

To bring any number of a lower denomination to the decimal of a higher denomination.

Divide it by the number of times one of the lower is contained in one of that higher denomination.

Therefore, to bring farthings to the decimal of a penny, divide them by 4.

To bring pence to the decimal of a shilling, divide them by 12.

To bring shillings to the decimal of a pound, divide them by 20.

To bring pounds to the decimal of any number of pounds, divide by that number.

To bring farthings to the decimal of a pound, divide their number at once by 960.

And generally to bring any concrete number to the decimal of a second concrete of the same kind, reduce them both to the lowest denomination mentioned in either, and divide the first by the second.

Ex. (i.) Reduce 2*s.* $6\frac{1}{2}$*d.* to the decimal of £1.

Here $\frac{1}{2}d. =$ two farthings $= \frac{2}{4}d. = {\cdot}5d.$

$$6\tfrac{1}{2}d. = 6{\cdot}5d. = \tfrac{6{\cdot}5}{12}s. = {\cdot}541\dot{6}s.$$

$$2s.\ 6\tfrac{1}{2}d. = 2{\cdot}541\dot{6}s. = £\tfrac{2{\cdot}541\dot{6}}{20} = £{\cdot}12708\dot{3}.$$

The operation is usually performed thus :—

$$\begin{array}{r|l} 4 & 2{\cdot} \\ \hline 12 & 6{\cdot}5d. \\ \hline & 2{\cdot}541\dot{6}s. \\ \hline £ & {\cdot}12708\dot{3} \\ \hline \end{array}$$

Ex. (ii.) Reduce 2*s.* $6\frac{1}{2}$*d.* to the decimal of 13*s.* $10\frac{1}{2}$*d.*

$$2s.\ 6\tfrac{1}{2}d. = \tfrac{61}{24}s.;$$

$$13s.\ 10\tfrac{1}{2}d. = \tfrac{111}{8}s.$$

Therefore, the fraction required $= \frac{61}{24} \div \frac{111}{8} = \frac{61}{333}$.
Consequently, the decimal fraction required $= \cdot\dot{1}8\dot{3}$.

The same may be found thus :—

$$\begin{array}{rl} 2s.\ \ 6\tfrac{1}{2}d. = 122 & \text{farthings.} \\ 13s.\ 10\tfrac{1}{2}d. = 666 & \quad ,, \end{array}$$

Therefore, the fraction that the first is of the second

$$= \frac{122}{666} = \frac{61}{333} = \cdot\dot{1}8\dot{3}.$$

Ex. (iii.) Reduce 3*s*. 4*d*. to the decimal of £3 15*s*.

$$\begin{array}{r|l} 12 & 4\cdot d. \\ \hline £3\ 15 = 75s.\ \therefore\ 75 & 3\cdot\dot{3}s. \\ \hline & \cdot0\dot{4} \\ \hline \end{array}$$

Ex. (iv.) Reduce 2 qrs. 4 lbs. to the decimal of a ton.

$$\begin{array}{r|ll} 28 & 4\cdot & \text{lbs.} \\ \hline 4 & 2\cdot\dot{1}4285\dot{7} & \text{qrs.} \\ \hline 20 & \cdot535714 & \text{cwt.} \\ \hline & \cdot026785 & \text{ton.} \\ \hline \end{array}$$

Ex. (v.) Find the value of ·0045 of £2.

$$\begin{array}{r} \cdot0045 \\ 2 \\ \hline £\cdot0090 \\ 20 \\ \hline \cdot1800s. \\ 12 \\ \hline 2\cdot1600 \\ \hline \end{array} \quad \text{Ans. } 2\cdot16d.$$

Ex. (vi.) Find the value of ·045 of 12*s*. 6¾*d*.

Here 12*s*. 6¾*d*. = 603 farthings.

$$\begin{array}{r} \cdot045 \\ \underline{603} \\ 135 \\ \underline{270} \\ 4)\underline{27\cdot135} \text{ farthings.} \\ \underline{6\cdot78375d.} \end{array}$$

Or by Vulgar Fractions.

$$\cdot045 = \frac{45}{1000} = \frac{9}{200}.$$

$$12s.\ 6\tfrac{3}{4}d. = \frac{201}{16}s.$$

$$\text{Therefore value} = \frac{201}{16}s. \times \frac{9}{200} = \frac{1809}{3200}s. = 6\cdot78375d.$$

21. To convert a vulgar fraction into a decimal. Add cyphers to the numerator and divide by the denominator. Art. 19.

Let $\frac{3}{8}$ be the vulgar fraction to be converted.

From the nature of fractions, 3 may be considered a dividend, and 8 its divisor.

Now 3 represents 3 units, and may be written 3·000. But 8 is not contained in 3, therefore 3 becomes the radix figure of the next denomination, and makes 30 tenths. 8 divides 30 tenths, and gives 3 tenths for a quotient, with remainder 6.

6 becomes the radix figure of the next denomination, which is hundredths; 8 divides 60 hundredths, and gives a quotient 7 hundredths, and remainder 4. This becomes the radix figure of the next denomination, which is thousandths.

8 divides 40 thousandths, and gives 5 thousandths for a quotient.

$$\therefore \quad 8\underline{)3{\cdot}000} \atop \underline{\;\;{\cdot}375}$$

22. *Addition.—Rule.* Place the numbers so that the decimal points shall be under one another, and thus form columns of the several denominations. Add, as in Simple Addition, putting the point under the column of points in the sum.

23. *Subtraction* is performed as in the subtraction of simple numbers, placing the numbers so that the points are under one another in the minuend, subtrahend, and difference.

24. *Multiplication.* Since units, tenths, hundredths, &c. multiplied by tenths, give tenths, hundredths, thousandths, &c. respectively for a product, and so on for other denominations, it follows that the product of two decimal numbers, or of numbers partly whole and partly decimal, will be found by multiplying as in Simple Multiplication, and then marking off from the product, if possible, as many figures as there are decimal places in the multiplicand and multiplier. If not possible, cyphers must be prefixed to make up the required number of decimal places.

Ex. $34{\cdot}56 \times 3{\cdot}456$.

Here the decimal part of the first is hundredths, and the decimal part of the second thousandths. Now hundredths multiplied by thousandths give hundred-thousandths; consequently there will be five figures in the decimal part of the result. This will be seen from the following.

The above numbers when expressed as vulgar fractions become respectively $\frac{3456}{100}$ and $\frac{3456}{1000}$.

Now $\frac{3456}{100} \times \frac{3456}{1000} = \frac{11943936}{100000} = 119{\cdot}43936$.

25. *Division.—Rule.* Make the number of decimal places in the divisor and dividend the same, by adding cyphers as required, *i.e.* bring them both to the same decimal denomination, and divide as in whole numbers. The quotient will consequently be a whole number, unless the dividend be less than the divisor. In this latter case cyphers must be added until it becomes equal to, or first greater than, the divisor; and, for every cypher thus taken after the first, a cypher must be put in the quotient after the decimal point.

Ex. Divide ·004 by 5.

Here the dividend is *four* thousandths, and divisor *five* units, and is, therefore, equivalent to five thousand thousandths.

Therefore ·004 ÷ 5 may be worked thus:

$$\begin{array}{l} 5000)\underline{4\quad\quad}(\cdot0008. \\ \quad\quad\ \ \underline{\cdot\ \cdot\ \cdot\ \cdot\ \cdot} \end{array}$$

5000 is not contained in 4, therefore there is no integer in the quotient.

„ „ „ 40 tenths, therefore ·0.

„ „ „ 400 hundredths, therefore ·00.

„ „ „ 4000 thousandths „ ·000.

is „ 40000 ten-thousandths, 8 times,

consequently the quotient is 8 ten-thousandths, or ·0008.

26. To convert a pure recurring decimal into a vulgar fraction.

Any figure with a cypher following, divided by 9, will leave a remainder equal to that figure; and also any number of two figures with two cyphers following, when divided by 99, will leave that number for a remainder. And so for any number whatever thus formed, and divided by a number consisting of as many nines as there are significant figures in the dividend. It follows from this, that a pure recurring decimal is converted into a vulgar fraction, by taking the repeating period for a numerator, and a number, consisting of as many nines as there are figures in the period, for a denominator.

27. To convert a mixed recurring decimal into a vulgar fraction. For a numerator, subtract the part not repeating from the number made up of the figures between the decimal point and end of the period; and, for a denominator, take a 9 for every figure repeating, followed by a cypher for every figure not repeating.

Ex. (i.) Take the decimal $\cdot 23\dot{4}\dot{5}$. This may be represented by the fractions $\frac{23}{100} + \frac{45}{9900} = \frac{2300}{10000} + \frac{45}{9900}$. If we consider the 2300 to be 9900$^{\text{ths}}$ instead of ten-thousandths, we take 23 ninety-nine hundredths too much. For the difference between $\frac{2300}{9900}$ and $\frac{2300}{10000} = \frac{2300}{9900} - \frac{2300}{10000} = \frac{2300}{100}\left(\frac{1}{99} - \frac{1}{100}\right) = 23\left(\frac{100-99}{9900}\right) = \frac{23}{9900}$. Consequently, by subtracting 23 from 2345 we get the number of ninety-nine hundredths, to which the decimal is equivalent.

Ex. (ii.) $\cdot 234\dot{5} = \frac{2340}{10000} + \frac{5}{9000}$, by considering the ·2340 as nine-thousandths, we take 234 nine-thousandths too much. Consequently the decimal is equivalent to the vulgar fraction, whose numerator is 2345—234, and denominator, 9000.

28. To compare recurring decimals, they must be first brought to equivalent vulgar fractions, with common denominators.

If the *period* be taken two or three times, addition and subtraction may be performed as in decimals not recurring, and the results will be sufficiently accurate for all practical purposes.

29. Decimal Notation in coinage, weights, measures, &c.

Suppose the coinage to be so adjusted that

£1 = 10 florins = 100 cents = 1000 mils,
1 florin = 10 cents = 100 mils,
1 cent = 10 mils,

and the same notation with different names adopted for weights, measures, &c., then all operations in Arithmetic would be conducted by the simple rules; *i. e.* they would involve only abstract numbers, or concrete numbers of one denomination.

Ex. (i.) £2. 3fl. 4c. 5m. = £2·345;
and 24 tons, 2 cwt. 3 qrs. 4lbs. = 24·234 tons.

Therefore the value of the latter at £2. 3 fl. 4 c. 5 m. each = £2·345 × 24·234. Thus, by simple multiplication the value is determined.

Ex. (ii.) What would be the cost of papering a wall 22 ft. 3′ 4″ long, 15 ft. 4′ 5″ broad, with paper 2 ft. 2′ 1″ broad, at 2 fl. 3 c. 4 m. per foot?

In these questions each denomination is supposed to be one-tenth of the next higher.

Therefore, 22·34 × 15·45 = area to be papered; and $\frac{22{\cdot}34 \times 15{\cdot}45}{2{\cdot}21}$ = number of linear feet of paper; consequently price $= \frac{22{\cdot}34 \times 15{\cdot}45}{2{\cdot}21} \times 2{\cdot}34$ florins

$$= \frac{22{\cdot}34 \times 15{\cdot}45}{2{\cdot}21} \times £{\cdot}234 = £36{\cdot}5456$$

$$= £36.\ 5\ \text{fl.}\ 4\ \text{c.}\ 5\ \text{m.}$$

Ex. (iii.) What is the net value of 24 boxes, each 2 cwt. 3′ 4″; tare 2′ 3″ per cwt.; tret 3″ per 100″; draft 2″ per 300″; at 1 fl. 4 c. per 1″?

Net value = $2{\cdot}34 \times 24 \times {\cdot}77 \times {\cdot}97 \times {\cdot}99\dot{3} \times £{\cdot}14$.

Because 2′·3″ per cwt. $= \frac{23}{100} = {\cdot}23$, we have ∴ to subtract from every cwt. this portion; thus 1·00 − ·23 = ·77; so 3″ per 100″ is 1·00 − ·03 = ·97. Similarly for 2″ per 300″ we have $1 - \frac{2}{300} = \frac{300}{300} - \frac{2}{300}$ ∴ $\frac{298}{300}$ remain.

Also the fraction $\frac{298}{300} = {\cdot}99\dot{3}$. And 1 fl. 4 c. = £·14.

CHAPTER V.

SQUARE ROOT.

1. To find the Square Root of a number.

1 is the square of 1
100 „ „ 10
10000 „ „ 100
1000000 „ „ 1000

Thus, we see if the number consist of one figure, or less than three, there will be only one figure in the root; if the number consist of three, or less than five, there will be two figures; if of five or less than seven, there will be three figures; if of seven or less than nine, there will be four figures &c. &c.; *i. e.* for every two figures after the first in the number, we shall have a figure in the root; and, if a point be placed over the units and every second figure after it, the number of points will be the number of figures in the root.

Also $\frac{1}{100}$ =·01 is the square of $\frac{1}{10}$; *i.e.* of ·1
„ $\frac{1}{10000}$ =·0001 „ $\frac{1}{100}$; „ ·01
„ $\frac{1}{1000000}$ =·000001 „ $\frac{1}{1000}$; „ ·001;
i. e. the square of a decimal will consist of twice as many decimal places as the decimal itself.

Rule. (i.) Find the square root of the number made up of the figures in the first period (reckoning from left to right): this will give the first figure in the root. Square this and subtract. To the remainder bring down the figures in the next period for a dividend.

(ii.) Find a trial divisor by doubling the quotient, and this will give the next figure in the root. Complete the divisor by annexing this figure to the trial divisor, then multiply the whole divisor by it, and subtract. Proceed as before till the root is obtained.

If the number consist of a whole number and decimal places, point the unit's figure, and every second figure to the left and right of it. If the number be a decimal only, place the first point over the second decimal figure, and then over every second figure to the right.

2. To find the *Fourth Root.* First find the square root. The square root of this will be the fourth root of the original number.

In finding the square root of very large numbers, or an approximate value of the root to a certain number of decimal places, the following method may be adopted.

First find one more than half the required number of figures of the root by the ordinary method, and then proceed for the rest as in common division, using the last divisor, and bringing down the remaining figures one by one. The proof of this is seen from the following:—

Let N be the number whose square root is required; and let *ten* be the required number of figures in the root.

Let a be the part of the root found by the usual method, and x be the part of the root required to be found.

$$\text{Then } N=(a+x)^2=a^2+2\,a\,x+x^2$$

$$\therefore\ \frac{N-a^2}{2a}=x+\frac{x^2}{2a},$$

$$\text{Now } x \text{ is} < 10^4 \text{ and } a \text{ is not} < 10^9$$

$$\therefore\ \frac{x^2}{2a} \text{ is} < \frac{10^8}{2\times 10^9} < \frac{1}{20} < 1,$$

and $\therefore$ the root may be found sufficiently exact, and if for 10 figures, so for any number.

Ex. Suppose the root to be 12345678.

Here $a=$ 12345,

Therefore the number represented by $a=$ 12345000,

and „ „ „ $x=$ 678.

3. To find the Cube Root of a number.

1	is the cube of		1
1000	„	„	10
1000000	„	„	100
1000000000	„	„	1000

&c. &c.

i. e. if the number consist of one figure, or less than four, the root will consist of one figure; if it consist of four figures, or less than seven, the root will consist of two figures; if it consist of seven figures or less than ten, the root will consist of three figures; if it consist of ten figures, or less than thirteen, the root will consist of four figures, &c.

If then the units' figure be pointed, and a point placed over every third figure to the left, the number of points will denote the number of figures in the root.

Rule. (i.) Find the nearest Cube Root of the number composed of the figures in the first period, reckoning from left to right, this will be the first figure in the root. Cube it, and subtract from the number, bringing down the figures in the next period to form with the remainder a new dividend.

(ii.) Find a *trial* divisor by taking three hundred times the square of the *quotient*; this will give the next figure in the root.

(iii.) Complete the *divisor* by adding to the trial divisor thirty times the product of the former part of the quotient, and the new figure just obtained, together with the square of the new figure.

(iv.) Multiply the divisor by this last figure of the quotient, and subtract the product from the partial dividend; then bring down the figures in the next period to the remainder, and proceed as before, until the root required is obtained.

Any trial divisor after the first can be found without the trouble of squaring the quotient, viz. by adding to the last complete divisor the numbers which were added to the previous trial divisor, together with the square of the last figure, affixing two cyphers to the sum; *or in* *short*, to the three last lines in the operation for completing the divisor, add the square of the last figure found in the root, and affix two cyphers; the following example will illustrate the work:—

Ex. Find the Cube Root of 1879080904.

				$\dot{1}87\dot{9}08\dot{0}90\dot{4}(1234$
				1
$1^2 \times 300$	=	300		879
$1 \times 2 \times 30$	=	60	(1)	
2^2	=	4	(2)	
		364	(3)	728
2^2	=	4	(4)	151080
2nd trial divisor		43200		
$12 \times 3 \times 30$	=	1080		
3^2	=	9		
		44289		132867
3^2	=	9		18213904
3rd trial divisor		4538700		
$123 \times 4 \times 30$	=	14760		
4^2	=	16		
		4553476		18213904

If there be a decimal part to the number whose Cube Root is required, place a point over every third figure to the left and right, beginning with the units. Cyphers, if required, may be added to the decimal part. The number of points over the whole part will show the number of figures in the integral part of the root.

When half and one more than half the number of figures in the root have been obtained, the rest may be found by dividing the last remainder by the last divisor, bringing down the remaining figures one by one. This will be seen from the following:—

Let N be the number, and 10 the required number of figures in the root.

Let $a =$ the part of the root found by the rule; *i. e.* the first six figures of it.

Let $x =$ the part to be found; *i. e.* the last four figures.

Then $N = (a + x)^3 = a^3 + 3a^2x + 3ax^2 + x^3$.

Therefore $\frac{N - a^3}{3a^2} = x + \frac{x^2}{a} + \frac{x^3}{3a^2}$

Now x is less than 10^4, and a is not less than 10^9.

Therefore $\frac{x^2}{a}$ is less than $\frac{10^8}{10^9}$ *i. e.* than $\frac{1}{10}$;

and $\frac{x^3}{3a^2}$, is less than $\frac{10^{12}}{3 \times 10^{18}}$ is less than $\frac{1}{3 \times 10^6}$;

consequently $\frac{x^2}{a} + \frac{x^3}{3a^2}$ is less than $\frac{1}{10} + \frac{1}{3 \times 10^6}$ and therefore less than 1.

By supposing the number of figures in the root to be greater or less than 10, it may be proved that $\frac{x^2}{a} + \frac{x^3}{3a^2}$ is less than 1, and therefore the rule is sufficiently established.

Obs. No attempt at a proof of the rule for square root or cube root has been made, because the proof (so called) generally given is but the explanation of the process for finding the square and cube roots of an Algebraical expression.

4. *Simple Practice.*—To find the value of any number of articles of one kind and one denomination, by means of the aliquot parts of £1 sterling.

40 yards @ £1 each are worth £40;
but ,, ,, 16*s.* 6½*d.* each cannot be determined by one operation, because 16*s.* 6½*d.* is not an aliquot part of £1. The value, however, of 40 yards at 16*s.* 6½*d.* per yard is equivalent to the sum of the values of

40 yards at 10*s.*
40 ,, 5*s.*
40 ,, 1*s.*
40 ,, 6*d.*
40 ,, ½*d.*

These are represented thus in Practice:—

Ex. (i.)

	£	£	*s.*	*d.*	
10*s.*	$\frac{1}{2}$	40	0	0	= price of 40 yds. @ £1 ⅌ yd.
5*s.*	$\frac{1}{2}$	20	0	0	= ,, ,, @ 10*s.* ,,
1*s.*	$\frac{1}{5}$	10	0	0	= ,, ,, @ 5*s.* ,,
6*d.*	$\frac{1}{2}$	2	0	0	= ,, ,, @ 1*s.* ,,
½*d.*	$\frac{1}{12}$	1	0	0	= ,, ,, @ 6*d.* ,,
			1	8	= ,, ,, @ ½*d.* ,,
		£33	1	8	= ,, ,, 16*s.* 6½*d.*

Ex. (ii.) What is the value of 345⅜ yards @ £2 9*s.* 11*d.* ⅌ yard?

345 yards at £1 per yard are worth £345.
⅜ yard at £1 per yard is worth ⅜ of £1
= 7*s.* 6*d.*

	£	£	*s.*	*d.*	
6/8	⅓	345	7	6	=price of 345⅜ yds. @ £1 ⅌ yd.
2/6	⅛			2	
		690	15	0	= ,, @ £2 ,,
		115	2	6	= ,, @ 6*s.* 8*d.* ,,
6*d.*	⅕	43	3	5·25	= ,, @ 2*s.* 6*d.* ,,
3*d.*	½	8	12	8·25	= ,, @ 6*d.* ,,
		4	6	4·125	= ,, @ 3*d.* ,,
		£861	19	11·625	= ,, @ £2 9*s.* 11*d.* ,,

6*s.* 8*d.* is ⅓ of £1; therefore the price of 345⅜ yds. at 6*s.* 8*d.* is ⅓ of that at £1.

So the price at 2*s.* 6*d.* is ⅛ of the same.
,, ,, 6*d.* is ⅕ of that at 2*s.* 6*d.*, because 6*d.* is ⅕ of 30*d.*
,, ,, 3*d.* is ½ of that at 6*d.*

5. *Compound Practice—i. e.* when the quantities are concrete, but of different denominations.

Ex. (iii.) Find the value of 2 cwt. 3 qrs. 4 lbs. @ £2 16*s.* 6*d.* ⅌ cwt.

qrs.	cwt.	£	s.	d.	
2	$\frac{1}{2}$	2	16	6	=price of 1 cwt.
				2	
		5	13	0	= „ 2 cwt.
1	$\frac{1}{2}$	1	8	3	= „ 2 qrs.
4lb.	$\frac{1}{7}$		14	1·5	= „ 1 qr.
			2	0·214	= „ 4 lbs.
		£7	17	4·714	= „ 2 cwt. 3 qrs. 4 lbs.

6. *Duodecimals* comprise all operations in which the radix of notation is 12.

7. *Notation Scales.*—As in the common or ordinary system we can form any number by means of 10 for a radix; so by taking any other radix we can form a series of numbers equivalent to others calculated to the common radix.

The number 2345, when the radix is 10, represents $2 \times 10^3 + 3 \times 10^2 + 4 \times 10 + 5$.

If 6 were the radix, the number 2345 would represent $2 \times 6^3 + 3 \times 6^2 + 4 \times 6 + 5$, and so for any other radix.

8. To transform a number expressed in one scale to its equivalent in any other.

In the common system (radix 10), when we have a number of units consisting of two figures, we say there are so many tens, and so many units remaining. So with a number of tens consisting of two figures we take out the hundreds, and put down the tens, and the same for the other denominations. For any other base or radix we must proceed in a similar manner. *Ex. gr.* 432 is a number (radix 10); if we wish to find its equivalent when the radix is 6, we must find how many times it contains 6.

Thus 6/432
6/72—0 *i. e.* there are 72 times 6 exactly, and no units remaining. The *sixes* now occupy the same place as the tens in the 10 scale.
6/12—0 Or there are 12 times 6^2, and none remaining.
2—0 " 2 " 6^3, " "

Therefore 432 in the ten scale = 2000 in the six scale, *i. e.* $= 2 \times 6^3 + 0 \times 3^2 + 0 \times 3 + 0$.

(i.) Again, 2345 in the (10) scale is transformed into the (8) scale in the same way, *i. e.* by dividing by 8.

8\ 2345
8\ 293—1 *i. e.* there are 293 times the radix, and 1 unit remaining.
8\ 36—5 " 36 " $(\text{radix})^2$ " 5 (radix) "
\ 4—4 " 4 " $(\text{radix})^3$ " 4 $(\text{radix})^2$ "

Therefore 2345 in the 10 scale = 4451 in the 8 scale.

(ii.) Transform 2345 from the 8 scale to the 6 scale.

6\2345 $= 2 \times 8^3 + 3 \times 8^2 + 4 \times 8 + 5$.
6\320—5
6\42—4
\5—4

Therefore 2345 in the 8 scale = 5445 in the 6 scale. 6 is not contained in 2, therefore 2 becomes the radix figure to 3, and we have $2 \times 8 + 3 = 19$ for a dividend (by the rule for division). There are 3 times 6 in 19, and 1 remaining, which becomes the radix figure for 4, and we get $1 \times 8 + 4 = 12$ for a dividend, and so on.

9. Examples in the ordinary rules will be given. from which the mode of applying a different radix will be seen at once.

Ex. 2345 to be added in the 13 scale. $t = 10.$
3478 $e = 11.$
98*t*4 $f = 12.$
*e*521
208*e*5 from this subtract *ef* 14.
*ef*14
119*t*1

In the above continued example the first column of the addition amounts to 18 in the decimal scale, which are equal to *one* thirteen and 5 units. The 5 is put down, and *one* carried. So for the other columns.

In the subtraction, 4 from 5, there remains 1.
1 „ 11, there remain 10.
12 „ 8, we cannot; borrow 13.
then 12 „ 21, there remain 9.
11 „ 0, we cannot; borrow 13.
11 „ 13, there remain 2.
1 „ 2, there remains 1.

Multiply 2345 by 47, and divide the product by 342 in the 8 scale.

2345
47
21103
11624
342)137343(330
1246
.1254
1246
...$\frac{63}{342}$

In the multiplication :—

7 × 5 = 35 = 4 × 8 + 3; *i. e.* = 4 times the radix and three units. Therefore we put down the 3, and carry the 4; and so for the rest.

To find the quotient, we multiply the last two figures to the left of the divisor by trial figures. In the above 34 × 4 will give 160, because 4 × 4 = 16 = 2 × 8 + 0 = twice the radix; 3 × 4 = 12, and 2 carried = 14 = once the radix + 6.

In this way all the other operations are performed.

To find the square root of 104524 in the 6 scale.

104524(234
4
43 / 245
213
504 / 3224
3224
....

The cube root is performed in the same manner as in the ordinary scale, attention being paid to the radix.

Obs.—In the ordinary scale the numeration table proceeds downwards from units to tenths, hundredths, &c.; in the duodecimal scale it proceeds downwards from units to twelfths, hundred and forty-fourths, &c.; and similarly for other scales.

Before we can multiply feet, inches, &c., &c. by a number of feet, inches, &c., &c. in the duodecimal scale, we must express the number of feet in the multiplier and multiplicand in the duodecimal scale, as we do decimals in the 10 scale.

To show this fully we shall work out the same example by the old method, and by the one just mentioned.

Let it be required to multiply 23 ft. 7 in. $5''$ by 42 ft. 3 in. $8''$.

	ft.	in.	$''$		
(i.)	23	7	5		
	42	3	8		
	991	11	6		
	5	10	10	$3'''$	
	1	3	8	11	$4''''$
Sq. ft.	999	2	1	2	4

(ii.) 23 = 1 *e* in the 12 scale (*e* being eleven).
42 = 36 „

$$\begin{array}{r} \text{ft.}\\ 1e\cdot75\\ 36\cdot38\\ \hline 138e4\\ 5tt3\\ e986\\ 5tt3\\ \hline 6e3\cdot2124\\ \hline \end{array}$$

Now 6*e*3 in the 12 scale = 999 in the 10 scale.

Therefore the answer is 999 sq. ft. 2, 1, 2, 4 as before.

10. Nearly all questions in arithmetic may be solved by one, or both, of the following methods:

(i.) By a proportion.

(ii.) By finding the *unit* of value, interest, measure, weight, &c. as the case may require.

11. The first method is usually denominated 'The Rule of Three,' or 'The Double Rule of Three.' One rule, however, will be enunciated to include all questions of this class.

Obs.—An effect must be produced by some agent or agents, which we shall call '*agency*.'

A different agency (but of the same kind) will produce a different but a proportionate effect (and of the same kind).

If there be several factors in an effect, they must be combined; so also the factors of the respective agencies.

Thus there will result four things or terms, of which three will be completely known; the fourth will not be known, but can be determined by means of the other three.

Two of these three will be of the same kind, and will form a ratio such that whatever multiple, part, or parts the antecedent is of the consequent, the same multiple, part, or parts must the unknown term be of the third known term. As the right result depends upon the correct formation of this ratio, we shall call it the *Fundamental Ratio*, solely for the purpose of marking its importance.

Rule.—The required term is found by multiplying the *fundamental ratio* by the third known term.

Ex. (i.) If 2 cwt. 3 qrs. 12 lbs. cost £3 15*s.*, what will be the cost of 1 ton 5 cwt. at the same rate?

As the cost of 1 ton 5 cwt. must be greater than that of 2 cwt. 3 qrs. 12 lbs. at the same rate, the antecedent of the *fundamental ratio* must be greater than the consequent; and therefore the *fundamental ratio* is $\frac{\text{1 ton 5 cwt. reduced to lbs.}}{\text{2 cwt. 3 qrs. 12 lbs. reduced to lbs.}}$ *i. e.* the *fundamental ratio* $= \frac{2800}{320}$.

Consequently the cost required $= £3\ 15s. \times \frac{2800}{320}$

$= £\frac{15}{4} \times \frac{280}{32}$

Cancelling, &c. $= £\frac{525}{16}$

$= £32\ 16s.\ 3d.$

Ex. (ii.) How many yards of cloth $\frac{3}{4}$ yd. wide will cover a wall whose surface contains 80 yards of paper an English ell wide?

As 3 quarters is a less quantity than 5 quarters, the answer must be greater than 80.

Therefore the *fundamental ratio* is $\frac{5}{3}$, and the number of yards of cloth $= \frac{5}{3} \times 80 = 133\frac{1}{3}$ yards.

Ex. (iii.) 1000 men have provisions for 85 days; after 17 days 150 men go away. How long will the provisions left serve the remainder?

After 17 days there are provisions for 1000 men for 68 days; and it is clear that they will last 850 men longer than 1000 men.

Therefore the *fundamental ratio* is $\frac{1000}{850}$, and number of days required $= \frac{1000}{850} \times 68 = 80$ days.

Double Rule of Three.

N.B.—If agency is to be determined, then the two sets of effects will form the *fundamental ratio*, and conversely.

By putting a symbol for the unknown quantity, the two ratios will be complete.

Ex. (i.) 5 men reap a rectangular field 800 ft. long and 700 ft. broad in $3\frac{1}{2}$ days of 14 hours each; in how many days of 12 hours each can 7 men reap a field 1800 ft. long and 960 ft. broad?

The factors of the first agency combined

$$= 5 \times 3\tfrac{1}{2} \times 14$$

The factors of the second agency combined

$$= x \times 12 \times 7$$

The factors of the first effect combined $= \ 800 \times 700.$
„ „ second „ $= 1800 \times 960.$

It is clear that the number of days must be greater than before.

Therefore the *fundamental ratio* is $\dfrac{1800 \times 960}{800 \times 700}$

$$\text{and } x \times 12 \times 7 = \frac{1800 \times 960}{800 \times 700} \times 5 \times 3\tfrac{1}{2} \times 14.$$

$$\text{Therefore} \quad x = \frac{1800 \times 960}{800 \times 700} \times \frac{5 \times 3\frac{1}{2} \times 14.}{12 \times 7}$$

$$= 9 \text{ days.}$$

Ex. (ii.) If the sixpenny loaf weighs 4·35 lbs. when wheat is at 5·75*s.* a bushel, what weight can be bought for $18{\cdot}1\dot{3}$*s* when wheat is at 18·4*s.* per bushel?

Here 6*d.* is the 1st agency, and 4·35 lbs. × 5·75*s.* = 1st effect
$18{\cdot}1\dot{3}$*s.* „ 2nd „ and x × 18·4*s.* = 2nd „

Therefore the *fundamental ratio* is $\dfrac{18{\cdot}1\dot{3}}{\frac{1}{2}}$

$$x \times 18{\cdot}4 = \frac{18{\cdot}1\dot{3}}{\frac{1}{2}} \times 4{\cdot}35 \times 5{\cdot}75.$$

$$\text{And } x = \tfrac{272}{15} \times \tfrac{2}{1} \times \tfrac{435}{100} \times \tfrac{575}{100} \times \tfrac{10}{184} = 49{\cdot}3 \text{ lbs.}$$

Ex. (iii.) If 10 men reap a field of $7\frac{1}{2}$ acres in 3 days of 12 hours each, how long will it take 8 men to reap 9 acres, working 16 hours a day?

The first agency = 10 × 3 × 12.
second „ = 8 × 16 × days required.
first effect = $7\frac{1}{2}$ acres.
second „ = 9 „

The *fundamental ratio* $=\frac{9}{7\frac{1}{2}}=\frac{18}{15}$

Therefore days required $\times 8\times 16=\frac{18}{15}\times 10\times 3\times 12$.

And days required $=\frac{18}{15}\times\frac{10\times 3\times 12}{8\times 16}$

$= 3\frac{3}{8}$ days of 16 hours, or 3 days 6 hours.

Ex. (iv.) If 100 men drink £20 worth of wine at 4*s*. 6*d*. per bottle, how many will drink £72 worth at 5*s*. per bottle in the same time?

First agency $= 100\times\frac{9}{2}s.$
Second „ = No. of men required × 5.
First effect = £20.
Second „ = £72.

Therefore the *fundamental ratio* $=\frac{72}{20}$, because more men are required.

The number of men required $\times 5=\frac{72}{20}\times\frac{900}{2}$, consequently the number of men

$$=\frac{72}{20}\times\frac{900}{2}\times\frac{1}{5}=324.$$

Ex. (v.) What is the weight of a block of stone 12 ft. 6 in. long, 6 ft. 6 in. broad, and 8 ft. 3 in. deep, when a block of the same stone 5 ft. long, 3 ft. 9 in. broad, and 2 ft. 6 in. deep, weighs 7500 lbs.?

First agency $=\frac{25}{2}\times\frac{13}{2}\times\frac{33}{4}$.
Second „ $= 5\times\frac{15}{4}\times\frac{5}{2}$.
First effect is required.
Second „ = 7500 lbs.

Therefore, the *fundamental ratio* $=\frac{\frac{25}{2}\times\frac{13}{2}\times\frac{33}{4}}{5\times\frac{15}{4}\times\frac{5}{2}}$

because the stone will be heavier.

No. of lbs. $=\frac{25}{2}\times\frac{13}{2}\times\frac{33}{4}\times\frac{1}{5}\times\frac{4}{15}\times\frac{2}{5}\times 7500$
$=107250$ lbs.
$=47$ tons 17 cwt. 2 qrs. 10 lbs.

Obs.—If the pupil has any difficulty in discovering at sight from the question, whether the answer is to be more or less than the effect or agency given (as the case may be), in order to determine the *fundamental ratio*, he can always resort to the following rule :—The first agency multiplied by the second effect=the second agency multiplied by the first effect.

Ex. (vi.) Divide 462 into four parts that shall be to one another as the numbers 4, 5, 6, 7.

There will be 22 equal parts in the whole, of which the first number contains 4, the second 5 ; and so on.

Therefore, first number $=\frac{4}{22}\times 462=84$.
second „ $=\frac{5}{22}\times 462=105$.
third „ $=\frac{6}{22}\times 462=126$.
fourth „ $=\frac{7}{22}\times 462=147$.

Ex. (vii.) Find a number to which 24 shall have the same ratio that 6 has to 10.

The number is to be greater than 24.

Therefore *fundamental ratio* $=\frac{10}{6}$.
and number required $=\frac{10}{6}\times 24=40$.

Ex. (viii.) Three numbers are to one another as 7, 8, 12. The least is 28 ; find the others.

Of such parts as the least contains 7, the second contains 8, and the third 12.

28 contains 7 four times.

Therefore second number $=4\times 8=32$;
And third „ $=4\times 12=48$.

12. Four numbers are in proportion when the product of the greatest and least equals that of the other two.

Therefore, to find a fourth proportional to three given numbers,

Divide the product of the second and third by the first.

13. Three numbers are proportional if the product of the extremes be equal to the square of the mean.

Therefore, to find a mean proportional between two numbers,

Take the square root of their product.

14. To find a third proportional to two numbers,

Divide the square of the second by the first.

15. *Questions in Tare and Tret.* To find the net weight.

Rule. (i.) Find what fraction of the gross is allowed for tare, and deduct it.

(ii.) From this remainder deduct the portion allowed for tret.

(iii.) And from what still remains deduct the weight allowed for draft; and so for all other deductions. The result will be the net weight.

N.B.—If the allowance in either case be independent of the gross weight, the former must first be subtracted from the latter.

Ex. (i.) What is the net weight of 10 boxes, each 2 cwt. 3 qrs. 25 lbs.; tare 3 lbs. per box; tret $2\frac{1}{2}$ lbs. in 100 lbs.; draft 1 lb. per box?

Deducting 4 lbs. from the gross of each box, we at once take off the tare and draft.

The portion allowed for tret will be $\frac{5}{200}=\frac{1}{40}$;
$\therefore$ $\frac{39}{40}$ will remain.

2 cwt. 3 qrs. 21 lbs. = remainder after deducting tare and draft $=\frac{47}{16}$ cwt.

Therefore net weight of the 10 boxes
$=\frac{47}{16}$ cwt. $\times\frac{10}{1}\times\frac{39}{40}=\frac{1833}{64}=$ 28 cwt. 2 qrs. $15\frac{3}{4}$ lbs.

Ex. (ii.) What is the net weight and value at 6*d.* per lb. of 24 boxes of goods, each 2 cwt. 3 qrs. 12 lbs.; tare allowed at 16 lbs. per cwt., tret at 4 lbs. in 104 lbs., draft at 2 lbs. per 3 cwt.?

16 lbs. per cwt. $=\frac{1}{7}$ cwt.; therefore $\frac{1}{7}$ of the gross weight is allowed for tare, and $\frac{6}{7}$ remain when tare is deducted.

4 lbs in 104 lbs. $=\frac{1}{26}$; therefore $\frac{25}{26}$ of $\frac{6}{7}$ of the gross remain when the tare and tret are deducted.

2 lbs. in 3 cwt. or 336 lbs. $=\frac{1}{168}$; and therefore $\frac{167}{168}$ of $\frac{25}{26}$ of $\frac{6}{7}$ of the gross remain when all deductions are made.

Now the gross weight of 1 box is 2 cwt. 3 qrs. 12 lbs.
$=2\frac{6}{7}=\frac{20}{7}$ cwt.

Therefore the net weight of 24 boxes
$=\frac{20}{7}$ cwt. $\times\frac{24}{1}\times\frac{6}{7}\times\frac{25}{26}\times\frac{167}{168}=56$ cwt. 0 qrs. 19·9 lbs.
Net value $=£\frac{20}{7}\times\frac{24}{1}\times\frac{6}{7}\times\frac{25}{26}\times\frac{167}{168}\times\frac{112}{1}\times\frac{6}{240}$
$=£\frac{100200}{637}=£157$ 6*s.* nearly.

Obs.—In solving questions in Tare and Tret, the pupil is only required to show the two following lines of work—one for the net weight, and the other for the net value, in case it is required :—

Net weight $=\frac{20}{7}$ cwt. $\times\frac{24}{1}\times\frac{6}{7}\times\frac{25}{26}\times\frac{167}{168}=$ &c.
Net value $=£\frac{20}{7}\times\frac{24}{1}\times\frac{6}{7}\times\frac{25}{26}\times\frac{167}{168}\times\frac{112}{1}\times\frac{6}{240}=$ &c.

16. To find the *Simple Interest* of any sum of money for a given time at a given rate per cent. per annum.

(i.) Find the *unit* of interest; *i. e.* the interest of £1 for 1 year at the given rate per cent., and multiply this by the time.

(ii.) Multiply the interest of £1 for the given time by the principal, considered as an abstract number.

N.B.—The *unit* can always be determined at sight, thus—

If the rate per cent. be 4, *the unit* $=£\frac{4}{100}=£\frac{1}{25}$.
" " $2\frac{1}{2}$ " $=£\frac{5}{200}=£\frac{1}{40}$.
" " $2\frac{5}{9}$ " $=£\frac{23}{900}$.
&c., &c.

17. To find the *Amount*—the principal, rate per cent., and time being given.

The *unit* is the amount of £1 for the given time.

Rule.—Multiply the *unit, i. e.* the amount of £1 for the given time, by the principal.

18. To find the *Principal*—the amount, rate per cent., and time being given.

Here, since £1 of principal produces the amount of £1 in the given time, the *unit* required will be the amount of £1 of principal for the given time.

Rule.—Divide the whole amount by the amount of £1 for the given time, considered as an abstract number.

19. To find the *Rate per Cent.*—the principal, amount, and time being given.

Here the *unit* is the interest of £1 *for the given time at* 1 *per cent.*

Rule.—Divide the whole interest by the interest of the principal for the given time at 1 per cent. considered as an abstract number.

20. To find the *Time*—the principal, amount, and rate per cent. being given.

Here the *unit* is the interest of £1 for 1 *year* at the given rate per cent.

Rule.—Divide the whole interest by the interest of the principal for 1 year at the given rate per cent.

Obs.—All questions in interest might also be solved by proportion. Examples in illustration are appended to this chapter.

21. *Compound Interest.*

The *unit* is the compound interest of £1 for the given time. This may always be put down at sight.

Rule.—Multiply the unit by the principal considered as an abstract number.

The following example is worked out in full, and explained—

Find the Compound Interest of £250 for 3 years at 5 per cent. per annum.

Proceeding according to the Definition 100, Chap. I.

The Interest of £1 for the 1st year $=£\frac{1}{20}$; the amount$=£\frac{21}{20}$

„ Principal of £1 „ 2nd „ $=£\frac{21}{20}$.

„ Interest „ „ „ „ $=£\frac{21}{20}\times\frac{1}{20}=£\frac{21}{20^2}$

„ Principal „ „ 3rd „ $=£\frac{21}{20}+\frac{21}{20^2}=£\frac{21^2}{20^2}$

„ Interest „ „ „ „ $=£\frac{21^2}{20^2}\times\frac{1}{20}=£\frac{21^2}{20^3}$

And so on for any number of years.

Therefore the compound interest of £1 for 3 years

$$=£\frac{1}{20}+\frac{21}{{}^{2}}+\frac{21^2}{20^3}$$

This is a Geometrical Progression, in which the common ratio is $\frac{21}{20}$.

Therefore the sum $=\frac{1}{20}\left(\frac{(\frac{21}{20})^3-1}{\frac{21}{20}-1}\right)=\frac{21^3}{20^3}-1.$

Note.—This method requires the pupil to know the expression for the sum of a series of numbers in Geometrical Progression.

Therefore the compound interest of £1 for 3 years =(the amount of £1 for the first year)^3−1.

Consequently the compound interest of £250 for 3 years at 5 per cent.$=\{(\frac{21}{20})^3-1\}\times$£250$=(\frac{9261}{8000}-1)$ £250$=$£$\frac{1261}{8000}\times\frac{250}{1}=\frac{1261}{32}=$£39 8 1·5.

Another method is the following:—

The interest for the 1st year is the simple interest of the principal;

The interest for the 2nd year is the simple interest of the principal, together with the interest of the first year's interest; and so on.

The work may be thus arranged—

To find the Compound Interest of £250 for 3 years, at 5 per cent. per annum.

Rule.—Multiply the principal by the rate per cent. and divide by 100.

$$
\begin{array}{lrlll}
 & 250 & & & \\
 & \underline{\cdot 05} & & & \\
(^1) & 12\cdot 50 & \therefore \text{ interest for 1st year} & & =£12\cdot 5 \\
 & \underline{\cdot 05} & & & \\
(^2) & \underline{\cdot 625} & & & \\
(^3) & 13\cdot 125 & \therefore \quad ,, & \text{2nd} ,, & =£13\cdot 125 \\
 & \underline{\cdot 05} & & & \\
(^4) & \underline{\cdot 65625} & & & \\
(^5) & 13\cdot 781 & \therefore \quad ,, & \text{3rd} ,, & =\underline{£13\cdot 781} \\
 & & \text{Whole interest} & & =£39\cdot 406 \\
 & & & & =£39\ 8s.\ 1\cdot 44d.
\end{array}
$$

(1) is the interest of principal for 1st year.
(2) „ the 1st year's interest.
(3) is found by adding (1) and (2) together.
(4) is the interest of the 2nd year's interest.
(5) is found by adding (3) and (4) together.

N.B.—To divide by 100 is the same as moving the decimal point two places to the left. Thus, to multiply 250 by 3·5, and divide by 100, is the same as multiplying 250 by ·035.

Also, in working questions by decimals, it is sufficient to retain three places of decimals in the results, taking care to increase the last decimal figure retained by 1, if the first figure rejected be 5 or greater than 5.

If the Compound Interest is required for a certain whole number of years, and a portion of the next year, (as, for instance, 3½ years), then the 4th year's interest must be found in the usual way, and one-half of it taken and added to the interest for the *three* years.

By Art. 21 the Compound Interest for any one of the years can be put down at sight thus:—Find the amount of £1 for the 1st year, express it by a fraction, raise the denominator to the power denoted by the number of years, and the numerator to a power one less.

Ex. Find the Compound Interest of £1 for the 10th year at 2½ per cent. per annum.

Interest of £1 for 1 year = £$\frac{5}{200}$ = £$\frac{1}{40}$; therefore the amount of „ „ = £1$\frac{1}{40}$ = £$\frac{41}{40}$.

Consequently the Compound Interest for the 10th

$$\text{year} = £\frac{41^9}{40^{10}}$$

Obs.—If A be the amount, P the principal, R the amount of £1 for 1 year, and n the years, then by the above $A=P\times R^n$, from which expression, when three of the quantities A, P, R, n, are given, the fourth can be found.

If the interest is to be paid half-yearly, quarterly, monthly, &c., then R will be the amount of £1 for half a year, a quarter, a month, &c. respectively, and n will be $2n$, $4n$, $12n$, &c. accordingly.

In general the time and rate per cent. can only be determined by employing logarithms.

22. To find the difference between Simple and Compound Interest.

Find the difference between the Simple and Compound Interest of £1 for the given time, and multiply it by the principal considered as an abstract number.

23. *Discount*—according to the Definition, Chap. I. Art. 101.

It follows that the discount has the same ratio to the interest for the given time as £1 to the amount of £1 for the given time.

Rule.—Divide the Simple Interest for the given time by the *amount* of £1 for the given time, *considered as an abstract number.*

24. For the *Present Worth*, multiply the present worth of £1 by the principal *considered as an abstract number.*

As soon as the interest of £1 is obtained (and this is always found at sight) the Discount and Present Worth of £1 can be at once set down : *e. g.*—

What is the Discount and Present Worth of £1 for 9 months at 4 per cent per annum ?

Interest of £1 for 9 months = £ $\frac{3}{00}$.
Discount " " = £$\frac{3}{103}$ ∵ it = £$\frac{3}{100} \div \frac{103}{100}$.
P. W. " " = £$\frac{100}{103}$ ∵ it = £$1 - \frac{3}{103}$.

When then the interest of £1 is obtained,

The Discount will have the same numerator, but the sum of the numerator and denominator for *its* denominator.

The Present Worth will have the denominator for *its* numerator, but the same denominator as the Discount.

Suppose again £$\frac{1}{25}$ is the interest of £1 for the time,
Then Discount = £$\frac{1}{26}$
P. W. = £$\frac{25}{26}$

25. All questions of *Stocks*, *Barter*, *Profit and Loss*, &c. are solved either by Proportion, or by finding the *unit*.

Examples of each are worked out and explained in the next chapter.

26. *Equation of Payments.*—*Rule.* Find the whole interest arising from the various sums, or their equivalents, for one year, or month, or day, &c. as the case may be, at any rate per cent. ; then divide this by the interest of the entire amount of the bills or sums at the same rate, for one year, or month, or day, &c.

N.B.—By equivalent sums we mean, for instance, that £100 for 3 months will produce the same interest as £300 for one month.

CHAPTER VI.

EXCHANGE.

The Definitions 125, 126, 127, 128 should be referred to.

Arbitration of Exchange requires a fuller explanation than has been given in the *definition* of it.

When goods are exported from one country, and imported by another, then—in order to prevent the trouble, risk, and loss of interest, &c. by sending out *Coin*, or *Bullion*, or *Specie* (these terms are used indiscriminately) in payment—Bills for the amount are given.

Ex. gr. A in England sends out goods to the value of £10,000 to B at Bombay, and C at Bombay sends goods to D in England to the amount of £10,000. D draws a bill for £10,000, payable in London, and sends it in payment to C, B purchases the bill of C, and gives him coin for it, and then sends it back to A in payment for his goods. A presents the bill to D, and receives the money. Thus no coin leaves the country, and no risk is encountered if A, B, C, D are all honourable and solvent. This might be done in various ways. But suppose a number of merchants in different countries to be engaged in commerce one with the other, and suppose the exports from one country (M) to another (N) to be in excess of the imports from N to M, then bills drawn on N would be in excess, because not so much needed, and consequently would be at a discount: whereas bills on M would be probably at a premium. Thence we see the necessity of billbrokers to negociate these bills. Further, a bill drawn on one country may find its way into many others in the course of transmission, and may pass through the hands of several merchants who know nothing of it but as an article of commerce. So also, if a bill is dishonoured by the person on whom it is drawn, all who have been concerned with it will be liable in order; and thus it frequently happens that the failure of one commercial firm involves a great many others in a similar misfortune.

Examples will be found further on.

Examples. Interest.

Ex. 1. What is the Interest of £250 13*s.* 4*d.* for 3½ years, at 2¼ per cent. per annum?

Four modes of working this question will be given.

The old rule is to multiply the principal by the rate per cent., and that product by the time; then divide by 100.

(α)

	£	s.	d.
$\frac{1}{4}$	250	13	4
			2
	501	6	8
	62	13	4
$\frac{1}{2}$	564	0	0
			3
	1692	0	0
	282	0	0
	19,74	0	0
	20		
	14,80		
		12	
		9,60	

Answer £19 14*s.* 9·6*d.*

(β) By Decimals.

£250·666
2·25
1253330
501332
501332
563·99850
3.5
281999250
169199550
100)1973·994750
19.73994750
or £19.740 nearly
20
14·8
12
9·6

(γ) Interest of £1 for given time $=£\frac{9}{400}\times\frac{7}{2}$.
And „ £250 13 4 $=£250\frac{2}{3}=£\frac{752}{3}$
∴ Interest of Principal $=£\frac{9}{400}\times\frac{7}{2}\times\frac{752}{3}$.
=£19 14*s.* 9·6*d.*

(δ) By Proportion.

Here it is clear that £250 13*s.* 4*d.* must bear the same proportion to £100 that the interest of £250 13*s.* 4*d.*

does to the Rate per Cent. Or, in other words, that the *fundamental ratio* is formed from £250 13*s*. 4*d*. and £100, and is a ratio of greater inequality. See Definition 85.

The work will be as follows:—

$$\text{Fundamental ratio} = \frac{250\ 13s.\ 4d.}{100}$$

$$\therefore \text{ Interest required} = £\frac{250\frac{2}{3}}{100} \times 2\tfrac{1}{4}$$
$$= £\tfrac{752}{300} \times \tfrac{9}{4}$$
$$= £19\ 14s.\ 9{\cdot}6d.$$

These examples are given to show the student the advantage of working by fractions.

(γ) This method has the advantage of complete explanation.

Ex. 2. What *sum* will amount to £250 13*s*. 4*d*. in 3½ years, at 4 per cent. per annum?

The Interest of £1 for the time $= £\tfrac{4}{100} \times \tfrac{7}{2} = £\tfrac{7}{50}$.

" Amount " $= £1 + \tfrac{7}{50} = £\tfrac{57}{50}$.

(The pupil would be expected to put down the amount of £1 for the time at sight.)

Consequently by the rule the Principal $= £\tfrac{752}{3} \div \tfrac{57}{50}$.
$= £\tfrac{752}{3} \times \tfrac{50}{57}$.
$= £219\ 17s.\ 7{\cdot}9d.$

⁂ The question would be worked thus by the student:

The amount of £1 for the time $= £\tfrac{57}{50}$

$\therefore$ Principal $= £\tfrac{752}{3} \times \tfrac{50}{57}$
$= £219\ 17s.\ 7{\cdot}9d.$

Another mode of working this example is—

As Amount of £100 for the given time : whole Amount :: £100 : Principal.

But it is not so expeditious, nor does it explain itself so clearly.

Ex. 3. At what *Rate per Cent.* will £200 2*s.* 6*d.* amount to £250 6*s.* 8*d.* in 5 years?

Interest of £1 for the time at 1 per cent. $= £\frac{5}{100} = £\frac{1}{20}$

" Principal " " $= £\frac{1}{20} \times £\frac{1601}{8}$

$\because$ £200 2*s.* 6*d.* $= £200\frac{1}{8} = £\frac{1601}{8}$

the whole Interest = £250 6*s.* 8*d.* − £200 2*s.* 6*d.*

= £50 4*s.* 2*d.*

$= £50\frac{5}{24} = £\frac{1205}{24}$

Consequently the Rate per Cent. $= £\frac{1205}{24} \div \frac{1601}{160}$

$= £\frac{1205}{24} \times \frac{160}{1601}$

$= £\frac{24100}{4803}$

$= £5\frac{85}{4803}$

This has been worked out in full, but many steps may be saved without injury to the explanation.

Ex. 4. In what *time* will £50 amount to £83 6*s.* 8*d.*, at $2\frac{1}{2}$ per cent. per annum?

Interest of £1 for 1 year $= £\frac{2\frac{1}{2}}{100} = £\frac{5}{200} = £\frac{1}{40}$

" £50 for 1 year $= £\frac{1}{40} \times 50 = £\frac{5}{4}$.

The whole Interest = £83 6*s.* 8*d.* − £50

= £33 6*s.* 8*d.*

$= £33\frac{1}{3} = £\frac{100}{3}$.

$\therefore$ the Time $= \frac{100}{3} \div \frac{5}{4}$

$= \frac{100}{3} \times \frac{4}{5}$

$= 26\frac{2}{3}$ years.

The pupil would be required to work this sum thus:—

Interest of £50 for 1 year $= £\frac{5}{4}$
Whole interest $= £33\frac{1}{3}$
∴ Time $= \frac{100}{3} \times \frac{4}{5}$
$= 26\frac{2}{3}$ years

N.B.—If the time in any question is given in days only, or in years and months, &c. then the months and days must be brought to the fraction of a year, and the work will be the same.

Ex. gr. 3 years 50 days $= 3\frac{50}{365} = 3\frac{10}{73} = \frac{229}{73}$ years.
3 years 2 months $= 3\frac{1}{6} = \frac{19}{6}$ years.

Compound Interest.

The *two operations* of multiplying by a number, and dividing the result by 100, will be included in *one.* Ex. gr. Multiply 2·34 by 5, and divide by 100; the work would appear thus in full:—

$$\begin{array}{r} 2{\cdot}34 \\ \underline{5} \\ 100)\underline{11{\cdot}70} \\ \underline{{\cdot}1170} \end{array}$$

We see, then, to divide a number by 100 is the same as moving the decimal point *two places* to the left.

We might therefore have represented

$2{\cdot}34 \times 5 \div 100$ thus:—

$$\begin{array}{r} 2{\cdot}34 \\ \underline{{\cdot}05} \\ {\cdot}1170 \end{array}$$

And in the operations following, this will be understood.

Another example will explain further:—

2·34 × 1·23 ÷ 100.

If the first two be multiplied together, we should have four places of decimals in the product. Consequently, if we make six places of decimals, by moving the point two places to the left, we shall have multiplied by 1·23, and divide the result by 100. The work is arranged thus:—

2·34 or 2·34
1·23÷100 ·0123
702
468
234
$\cdot 288\cancel{8}\cancel{2}$
not ·28782
but ·288 striking off 82.

Ex. 5. Find the Compound Interest of £333 6*s*. 8*d*. for 3½ years, at 5 per cent. per annum.

This example will be worked out in full for the purpose of explaining the method:—

£333 6*s*. 8*d*.=£333⅓=£333·333

∴ £333·333
·05 ∵ ·05=5÷100

(1) $16\cdot666\cancel{6}\cancel{5}$ ∴ interest for 1st year=£16·667
·05

(2) ·83333 = interest on 1st year's interest.

$({}^1)+({}^2)=({}^3)$ 17·499 ∴ interest for 2nd year = £17·499
·05

$({}^4)$ ·87495 = interest on 2nd year's interest

$({}^3)+({}^4)=({}^5)$ 18·374 ∴ interest for 3rd year = £18·374
·05

$({}^6)$ ·91870

½)19·293

9·646 ∴ interest for ½-year = £9·646
whole interest = £62·186

In the line $({}^1)$ $\cancel{6}\cancel{5}$ are marked off, but in multiplying that line by ·05, this 6 was multiplied, and the 3 carried to the next product. In adding $({}^1)$ and $({}^2)$, only three places of decimals are reserved; and as the first figure omitted is 3, no addition to the first figure retained was necessary. In adding $({}^3)$ and $({}^4)$, the first figure omitted is 9; consequently 1 is added to the last figure retained. So also in adding $({}^5)$ and $({}^6)$ together the same allowance is made.

For the last half-year's interest, we found the interest for the 4th year, and took half of it; if the time had been 3 years and any other part of another year, we should have taken *that* part of the 4th year's interest. The whole interest for 3½ years is found by adding up the several items of interest on the right.

Another example will be given for the sake of further illustrating the work.

Ex. 6. Find the Compound Interest of £50 3*s*. 9*d*. for 3 years 73 days, at $3\frac{1}{8}$ per cent.

£50 3*s*. 9*d*. = £50·1875 = £50·188 nearly.
3 years 73 days = $3\frac{1}{5}$ years.
$3\frac{1}{8}$ = 3·125
$3\frac{1}{8} \div 100$ = ·03125 = ·031

£50·188
·031
50188
150564
(1) 1·556 ⋯ ∴ 1st year's interest = £1·556
·031
1556
4668
(2) ·048236 = interest on 1st year's interest
1·604 ⋯ ∴ 2nd year's interest = £1·604
·031
1604
4812
(3) ·049724 = interest on 2nd year's interest
1·654 ∴ 3rd year's interest = £1·654
·031
1654
4962
·051274 = interest on 3rd year's interest
$\frac{1}{5}$)1·705 = ·341

∴ $\frac{1}{3}$ of 4th year's interest =£ ·341

∴ whole interest =£5·155

£5·155
20
3·100
12
1·2

∴ Interest =£5 3*s.* 1·2*d.*

Discount and Present Worth.

The student should make himself familiar with the explanation under Articles 52, 53; the work will then be fully understood.

Ex. 7. What is the Discount and Present Worth of £25 3*s.* 4*d.* due 3 months hence, at 4 per cent. per annum?

The *Interest* of £1 for 3 months = £$\frac{1}{100}$

∴ Discount = £$\frac{1}{101}$

Present Worth = £$\frac{100}{101}$

∴ Discount of £25 3*s.* 4*d.* = £$\frac{1}{101} \times 25\frac{1}{6}$
= 4*s.* 11·8*d.*

Present Worth = £$\frac{100}{101} \times 25\frac{1}{6}$
= £24 18*s.* 4·1*d.*

Ex. 8. What is the difference between the True and Bankers' Discount on £25 3*s.* 4*d.* due 3 months hence, at 4 per cent. per annum? (See Definition 102.)

Bankers' discount on £1 for 3 months = £$\frac{1}{100}$
True " " " = £$\frac{1}{101}$
∴ Difference of " " " = £$\frac{1}{100} - \frac{1}{101}$
= £$\frac{1}{10100}$

∴ Difference of discount on £25 3*s.* 4*d.* $= £\frac{1}{10100} \times 25\frac{1}{6}$
$= £\frac{151}{60600}$
$= \frac{1}{2}d.$ nearly.

Stocks.

The student should clearly understand the difference between *Stock* and *Capital.*

All questions involving income may be solved as examples of Interest.

Ex. gr. What income arises from investing £200 in the 3 per cents. at 92? might be read—What is the Interest on £200 at 3 per 92? *i. e.* the Rate of Interest is £3 per £92. An example will be worked out below.

Ex. 9. How much Bank Stock in the $3\frac{1}{2}$ per cents. can be bought for £500 6*s.* 8*d.* at 92?

Here the *unit* is, how much stock can be bought for £1 capital?

£92 cash buy £100 stock
∴ £1 " $£\frac{100}{92}$ stock
∴ £500 6*s.* 8*d.* " $£\frac{100}{92} \times 500\frac{1}{3}$
$= £543$ 16*s.* 9·7*d.*

or thus, by Proportion,

There will be more stock than capital,

∴ the *Fundamental Ratio* is a ratio of greater inequality.

Fundamental Ratio $= \frac{100}{92}$
∴ Stock required $= £\frac{100}{92} \times 500\frac{1}{3}$
$= £543$ 16*s.* 9·7*d.*

Ex. 10. What will be the purchase of £2000 Stock in the $3\frac{1}{2}$ per cents. at 95 ?

Here the unit is, what will £1 Stock sell for ?

£1 Stock sells for $£\frac{95}{100}$

∴ £2000 „ $£\frac{95}{100}\times 2000$

$=£1900.$

Ex. 11. What Income arises from investing £2000 in the $3\frac{1}{2}$ per cents. at 85 ? or,

What is the Interest of £2000 at $3\frac{1}{2}$ per 85 ?

$$\text{Interest of } £1 = £\frac{3\frac{1}{2}}{85}=\frac{7}{170}$$

$$\therefore \text{Income} = £\frac{7}{170}\times 2000$$

$$=£82\ 7s.\ 0{\cdot}7d.$$

Ex. 12. Transferred £2000 Stock from the 3 per cents. at 85 to the 4 per cents. at 90. Find the change in income.

N.B. As this is a question of income, we shall work it as a question of Interest, and so save the clumsy mode of selling out of one stock and buying into another.

The Interest of £1 of capital in 3 per cents. $=£\frac{3}{85}$

„ „ „ 4 „ $=£\frac{4}{90}$

∴ gain on £1 of capital $=£\frac{4}{90}-\frac{3}{85}$

$=£\frac{7}{765}$

∴ whole gain $=£\frac{7}{765}\times(\frac{85}{100}\times 2000)$

$=£15\frac{5}{9}.$

The only difficulty is to find the Capital.

Now £1 Stock sells for $£\frac{85}{100}$

∴ £2000 „ $(£\frac{85}{100}\times 2000)$=the capital.

This may always be put down at sight.

The other mode is as follows:—

The Stock in the 4 per cents. will be less than £2000 in the 3 per cents., because the price of the 4 per cents. is the greater.

The Fundamental Ratio $= \frac{85}{90}$

∴ Stock in 4 per cents. $= £\frac{85}{90} \times 2000$

The Interest of Stock in 4 per cents. $= £\frac{85}{90} \times 2000 \times \frac{1}{25}$

(∵ Interest of £1 at 4 per cent. $= \frac{1}{25}$) $= £75\frac{5}{9}$

The Interest of Stock in 3 per cents. $= £2000 \times \frac{3}{100}$

$= £60$

∴ Change in income $= £75\frac{5}{9} - £60$

$= £15\frac{5}{9}$

Ex. 13. How much 4 per cent. Stock at $94\frac{1}{2}$ can be bought by the transfer of £200 13*s.* 4*d.* Stock in the 3 per cents. at $88\frac{3}{4}$?

(The question turns upon the *price* of the stocks, as income is not involved. We might have called the stocks by any other name instead of 4 per cent. and 3 per cent.)

4 per cent. Stock obtained by the transfer will be less than the amount of the 3 per cent. ∵ the price is greater.

∴ the Fundamental Ratio $= \frac{88\frac{3}{4}}{94\frac{1}{2}} = \frac{355}{378}$

∴ 4 per cent. Stock $= £\frac{355}{378} \times 200$ 13*s.* 4*d.*

$= £\frac{355}{378} \times 200\frac{2}{3}$

$= £188$ 9*s.* 1·6*d.*

Ex. 14. What amount of Stock in the $3\frac{1}{2}$ per cents. will produce *the same income* as £2316 13*s.* in the 3 per cents.?

The income depends on the *rate*, and ∴ the price is not needed, as the *income* is to be the *same*.

As less $3\frac{1}{2}$ per cent. Stock will be required,

$$\text{The Fundamental Ratio} = \frac{3}{3\frac{1}{2}}$$

$$\therefore\ 3\tfrac{1}{2} \text{ per cent. Stock} = £\frac{3}{3\frac{1}{2}} \times 2316\ 13s.$$

$$= £\tfrac{6}{7} \times 2316\tfrac{13}{20}$$

$$= 1985\ 14s.$$

The Examples 13, 14, should be carefully observed.

Ex. 15. The 4 per cents. being at $89\frac{1}{2}$, and the 5 per cents. at 108, which is the better Stock to buy?

The better Stock is that which gives the highest interest for £1.

$$\text{Interest of £1 of capital in the 4 per cent.} = £\frac{4}{89\frac{1}{2}} = \cdot 044$$

$$\text{,, ,, ,, 5 ,,} = £\tfrac{5}{108} = \cdot 046$$

∴ the 5 per cent. Stock is the better purchase.

Ex. 16. What rate of Interest arises from money invested in the 4 per cents. at $91\frac{3}{7}$?

(1) The question is, if £$91\frac{3}{7}$ realise an interest of £4, what will £100 realise?

$$\text{The Fundamental Ratio} = \frac{100}{91\frac{3}{7}}$$

$$\therefore \text{ Rate per cent.} = \frac{100}{91\frac{3}{7}} \times 4$$

$$= \tfrac{700}{640} \times 4$$

$$= 4\tfrac{3}{8}$$

$$\text{or (2) The Interest of £1} = £\frac{4}{91\frac{3}{7}}$$

∴ the Interest of £100 $= \pounds\frac{4}{91\frac{3}{7}} \times 100$
$= \pounds 4\frac{3}{8}$

Ex. 17. If the 4 per cents. pay an Interest of £4 12*s*. 6*d*. per cent. on the capital invested, what is the price of the Stock?

If £100 capital realise £4 12*s*. 6*d*. or £4$\frac{5}{8}$, then less than £100 will realise £4,

∴ the price is less than £100,

∴ the Fundamental Ratio $= \frac{4}{4\frac{5}{8}} = \frac{32}{37}$

∴ price of the Stock $= \pounds\frac{32}{37} \times 100$
$= \pounds 86\frac{18}{37}$.

Ex. 18. Bought Railway Shares at 10 per cent. discount; sold them at 5 per cent. premium, and gained £345 by the transaction; what sum did I invest? See Definitions 121, 122, 123.

A share cost £90, and sold for £105,

∴ the gain on one share = £15,

∴ the number of times that 15 is contained in 345 will be the number of shares bought,

∴ number of shares $= \frac{345}{15}$

Each share cost £90,

∴ sum invested $= \pounds\frac{345}{15} \times 90$
$= \pounds 2070$

In all transactions of this kind Brokers charge a commission, which must therefore be taken into account when it is specified.

Ex. 19. Suppose, for instance, in the last example the commission for buying had been £$\frac{1}{8}$ per cent., and another £$\frac{1}{8}$ per cent. had been charged for the stamp and registration in the books of the railway company, then every share would have cost £90$\frac{2}{8}$;

∴ the money invested would have been

$$=£\frac{345}{15} \times 90\frac{1}{4}$$
$$=£2075\ 15s.$$

So in all questions of buying and selling Stock, the commission must be added to the price of it, when specified.

Profit and Loss.

In all ordinary questions when the cost price is known, and the selling price is found, or *vice versâ*, the gain or loss is found by subtracting the sale from the cost, or the cost from the sale respectively.

Ex. 20. Bought 300 yards at 5*s.* per yard; sold 150 yards of them at 6*s.* 8*d.* per yard, and the rest at a loss of 10 per cent. How much did I gain or lose altogether?

The cost of 300 yards $=£\frac{1}{4} \times 300 = £75$

(1) The sale of 150 yards $=£\frac{1}{3} \times 150 = £50$

On the remaining 150 yards he lost $\frac{1}{10}$ of the cost;

∴ he sold them at $\frac{9}{10}$ of their cost.

(2) ∴ sale of remaining 150 yds. $=£\frac{1}{4} \times \frac{9}{10} \times 150$

$=£33\ 15s.$

The whole sale (1) and (2) $=£83\ 15s.$

∴ gain $=£83\ 15s. - £75$

$=£8\ 15s.$

Ex. 21. Bought an article at £2 15*s*. per cwt. and sold it at 10*d*. per lb. What was the gain per cent.?

Cost of 1 cwt. $=£2\ 15s.=£2\frac{3}{4}=£\frac{11}{4}$
Sale ,, $=£\frac{14}{3}$
$\therefore$ gain per cwt. $=£\frac{14}{3}-£\frac{11}{4}=£\frac{23}{12}$.
Now the cost, viz. $£\frac{11}{4}=£\frac{33}{12}$
$\therefore$ gain $=£23$ on $£33$
$\therefore$ the proportion gained $=\frac{23}{33}$ of the cost,
$\therefore$ gain per cent. $=£\frac{23}{33}\times 100$
$=£69\frac{23}{33}$

This Example is explained fully, to save the necessity of doing so in the following Examples.

Ex. 22. Bought an article for 12*s*. 6*d*., and paid $2\frac{1}{2}$ per cent. on the purchase for commission. I gained 20 per cent. by the sale. What did I sell it for?

The commission $=£\frac{5}{200}=\frac{1}{40}$ of the purchase,
$\therefore$ whole cost $=£\frac{5}{8}\times\frac{41}{40}$
The gain is 20 per cent. $=\frac{1}{5}$ of the cost,
$\therefore$ the sale $=£\frac{5}{8}\times\frac{41}{40}\times\frac{6}{5}$
$=15s.\ 4{\cdot}5d.$

Ex. 23. Sold goods for £2 10*s*. at a profit of $16\frac{2}{3}$ per cent. Find the cost.

The gain $=\frac{50}{300}=\frac{1}{6}$ of the whole cost;
$\therefore$ £2 10*s*. $=\frac{7}{6}$ of the cost,
$\therefore$ the cost $=£\frac{5}{2}\times\frac{6}{7}$
$=£2\ 2s.\ 10{\cdot}3d.$

Ex. 24. Sold an article for £25 10*s*. and lost 4 per cent. What is the gain or loss per cent. by selling it at £30?

Loss $=\frac{4}{100}=\frac{1}{25}$ of the cost,

∴ £25 10*s*. $=\frac{24}{25}$ of the cost ∵ $\frac{1}{25}$ is lost,

∴ the cost $=$ £25 10*s*. $\times\frac{25}{24}$

$=£\frac{425}{16}$

In the 2nd case the sale $=£30=£\frac{480}{16}$

∴ gain in the 2nd case $=£\frac{480}{16}-£\frac{425}{16}=£\frac{55}{16}$

or gain $=£55$ on £425 or $\frac{55}{425}$

∴ gain per cent. $=£\frac{55}{425}\times 100$

$=£12\frac{16}{17}$.

Ex. 25. A grocer mixed equal quantities of three kinds of tea, at 2*s*. 6*d*., 3*s*. 9*d*. and 4*s*. 9*d*. per lb. respectively. At what price per lb. must he sell the mixture to gain 10 per cent. ?

The cost of 3 lbs. of mixture $=12s.$

„ gain $=\frac{1}{10}$ of the cost,

∴ sale of 3 lbs. $=12s.\times\frac{11}{10}$

∴ „ 1 lb. $=12s.\times\frac{11}{10}\times\frac{1}{3}$

$=4s.\ 4{\cdot}8d.$

Ex. 26. What must I charge or ask to obtain 3*s*. 4*d*. for an article, after deducting 5 per cent. discount ?

I deduct $\frac{1}{20}$ of the charge,

∴ 3*s*. 4*d*. $=\frac{19}{20}$ of the charge,

and ∴ the charge $=3s.\ 4d.\times\frac{20}{19}$

$=3s.\ 6{\cdot}1d.$

Ex. 27. If an article cost 12*s*. what must I *charge* to gain $16\frac{2}{3}$ per cent. *on the cost*, after allowing a discount of $12\frac{1}{2}$ per cent. ?

The cost $=12s.$

The gain $=\frac{50}{300}=\frac{1}{6}$ of the cost,
$\therefore$ the sale $=12s.\times\frac{7}{6}=14s.$
I deduct $12\frac{1}{2}$ per cent., *i. e.* $\frac{25}{200}$ or $\frac{1}{8}$ from the charge;
Consequently $14s.=\frac{7}{8}$ of charge,
$\therefore$ charge $=14s.\times\frac{8}{7}$
$=16s.$

Partnership.

Ex. 28. A, B, and C speculate and gain £300. A advanced £200, B £150, C £180. What share of the gain will each have?

Here £200 + £150 + £180 or £530 gain £300, consequently £1 gains £$\frac{300}{530}$.

$\therefore$ £200 or A's money gains £$\frac{300}{530}\times 200=$ £113 4$s.$ 1·8$d.$
B's " £$\frac{300}{530}\times 150=$ £84 18$s.$ 1·3$d.$
C's " £$\frac{300}{530}\times 180=$ £101 17$s.$ 8·8$d.$

Ex. 29. A, B, C, join in partnership, and gain £250 6$s.$ 8$d.$ A advances £200 for 4 months, B £300 13$s.$ 4$d.$ for 6 months, C £150 6$s.$ 8$d.$ for 9 months. What is A's share of the gain?

£200 for 4 months = £800 for 1 month.
£300 13$s.$ 4$d.$ " 6 " = £1804 "
£150 6$s.$ 8$d.$ " 9 " = £1353 "
Now £800 + £1804 + £1353 = £3957
$\therefore$ £3957 gain £$250\frac{1}{3}$ in 1 month,

$$\therefore \text{A's share} = £\frac{250\frac{1}{3}}{3957}\times 800$$
$$= £50\ 12s.\ 2{\cdot}5d.$$

Ex. 30. A and B rent a field for £30. A puts into it 10 cows for 3 months; B 12 cows for 5 months, and 20 sheep for 2 months. What should each pay, supposing 5 sheep are equivalent to 1 cow?

A puts in 10 cows for 3 months=30 cows for 1 month.

$$\text{B} \quad ,, \left\{\begin{array}{l} 12 \quad\quad ,, \quad 5 \quad ,, \quad =60 \\ \left.\begin{array}{l}20\text{ sheep}\\ =4\text{ cows}\end{array}\right\} 2 \quad ,, \quad =\ 8 \end{array}\right\} \quad\quad ,,$$

$\therefore$ 98 cows for 1 month cost in keep £30

$\therefore$ A's share of rent $=£\frac{30}{98}\times 30=$ £9 3*s*. 8*d*.

B's „ $=£\frac{30}{98}\times 68=$ £20 16*s*. 3*d*.

Equation of Payments.

N.B. When Questions of this kind are independent of the rate of interest, we may use a symbol for it, *e. g.* let R.=Interest of £1 for 1 year, or 1 month, or 1 day, as the case may require; E. T. stands for the *Equated Time.*

Ex. 31. A person owes £200 payable in 3 months, £300 in 5 months, and £150 in 4 months. When should the whole be paid at once?

£200 in 3 mo. produce the same interest as £600 in 1 mo.
£300 „ 5 „ „ „ £1500 „
£150 „ 4 „ „ „ £600 „

$\therefore$ the whole interest=2700R.

The interest of the whole debt for 1 month=650R

$$\therefore \text{E. T.}=\frac{2700\text{R}}{650\text{R}}=4\tfrac{2}{13}\text{ months.}$$

N.B. The pupil would only be required to show two lines of this work, thus:—

Whole interest $=600\text{R}+1500\text{R}+600\text{R}=2700\text{R}$.

$$\therefore \text{E. T.} = \frac{2700\text{R}}{650\text{R}} = 4\tfrac{2}{13} \text{ months.}$$

Ex. 32. A debt of £500 is paid by monthly instalments of £10 each, and a ready-money payment of £200. Find the Equated Time.

There will be 30 payments: thus, £10 in one month, £10 in two months, &c.

∴ The whole interest $=10\text{R}+20\text{R}+30\text{R}+$ &c. to 30 terms $=(20\text{R}+29\times10\text{R})\,15$ by Arithmetical Progression $=4650\text{R}$

$$\therefore \text{E. T.} = \frac{4650\text{R}}{500\text{R}} = 9\tfrac{3}{10} \text{ months.}$$

Square Root.

Ex. 33. Find the Square Root of 6·173 to four places of decimals.

```
          ·6·17̇30̇              (2·4845
           4
          ───
  44    │ 217
   4   /  176
 ──────────────
  488  │  4130
    8  │  3904
 ───────────────
  4964 │  22600
     4 │  19856
 ────────────────
  49685 │  274400
        │  248425
        │  ──────
        │   25975
```

A new trial divisor in each case is obtained by adding to the previous divisor its last figure.

Ex. 34. Find the Cube Root of 355647·339125.

$35\dot{5}64\dot{7}\cdot33\dot{9}12\dot{5}(70\cdot85$

343

	$70^2 \times 300 = 1470000$	12647339
(1)	$70 \times 8 \times 30 = 16800$	
(2)	$8^2 = 64$	
(3)	1486864	11894912
(4)	64	752427125
2nd trial divisor	= 150379200	
$708 \times 5 \times 30$	= 106200	
5^2	= 25	
2nd complete div.	= 150485425	752427125
		....

A full explanation is given of the above process under Art. 32.

Exchange.

The course of Exchange between England and the various Continental countries is constantly changing; but an average value of the current coins is given merely as a guide to the Examples, and to save any further explanation.

France.—25 fr. 30 cents. = £1 sterling.
100 cents. = 1 franc.

America.—1 dollar = 4*s.* 6*d.* English money.

Hamburg.—13 marks 8 schillings = £1 English money. 16 schillings = 1 mark.

East Indies.—1 rupee = $23\frac{3}{4}$*d.*

Holland.—$11\frac{3}{4}$ florins =£1 English.

Ex. 35. Exchange £200 for French francs when English money bears a premium of £2 per cent.

In this case £1 is worth $\frac{1}{50}$ more, *i. e.* £$\frac{51}{50}$

∴ If 25 fr. 30 c. be the value of £1 at par,

25 fr. 30 c. $\times \frac{51}{50}$=£1 at a premium of 2 per cent.

and ∴ £200 =25 fr. 30 c. $\times \frac{51}{50} \times 200$.

=5161·2 francs.

If English money had been at £2 per cent. discount at Paris,

Then £1 is worth $\frac{1}{50}$ less than £1 or £$\frac{49}{50}$

∴ £200 „ 25 fr. 30 c. $\times \frac{49}{50} \times 200$

=4958·8 francs.

Ex. 36. Exchange 1000 American dollars for English money when it bears a premium of 20 per cent.

One dollar at par =4*s.* 6*d.*

but 4*s.* 6*d.* is worth $\frac{1}{5}$ more,

∴ one dollar + $\frac{1}{5}$ dollar =4*s.* 6*d.*=£$\frac{9}{40}$

or $\frac{6}{5}$ dollars =£$\frac{9}{40}$

∴ 1 dollar =£$\frac{9}{40} \times \frac{5}{6}$

1000 dollars =£$\frac{9}{40} \times \frac{5}{6} \times 1000$

=£187 10*s.*

Ex. 37. Exchange £250 for American dollars when English money is at a premium of 5 per cent.

English money is worth $\frac{1}{20}$th more,

∴ it will purchase $\frac{1}{20}$th more dollars.

4*s.* 6*d.* =£$\frac{9}{40}$

Now 4*s*. 6*d*. is worth 1 dollar $+\frac{1}{20}$ dollar $=\frac{21}{20}$ dollars,

$\therefore$ £$\frac{9}{40}$,, $\frac{21}{20}$ dollars,
and £1 ,, $\frac{21}{20}\times\frac{40}{9}$ dollars,
$\therefore$ £250 ,, $\frac{21}{20}\times\frac{40}{9}\times 250$ dollars,
$=1166{\cdot}6$ dollars.

Ex. 38. If the Exchange between England and New York be 4*s*. 6*d*. per dollar; between New York and Paris 5·4 francs per dollar; between Paris and Holland 115 francs per 55·5 florins; and between Holland and Hamburg $11\frac{3}{4}$ florins per 13·5 marks; find the Exchange between *London* and *Hamburg*, and the loss or gain in remitting £100 from London to Hamburg by the above route, and in sending it direct, if 13·5 marks = £1 sterling.

N.B. Questions of Arbitration are solved best by the Chain Rule.

£1 Eng. = 20*s*.
4*s*. 6*d*. or $\frac{9}{2}$*s*. = 1 dollar.
1 dollar = 5·4 francs.
115 francs = 55·5 florins.
11·75 florins = 13·5 marks.

Then all the terms on one side multiplied together will *equal* all those on the other side multiplied together, the first result being pounds, and the second marks; thus:

£$1\times\frac{9}{2}\times 1\times 115\times 11\frac{3}{4}=20\times 1\times 5\frac{4}{10}\times 55\frac{1}{2}\times 13\frac{1}{2}$ marks.

$\therefore$ £$1 = 20\times\frac{54}{10}\times\frac{111}{2}\times\frac{27}{2}\times\frac{2}{9}\times\frac{1}{115}\times\frac{4}{47}$.

$\because$ both sides are divided by $\frac{9}{2}\times 1\times 115\times 11\frac{3}{4}$

$=13{\cdot}3$ marks nearly.

PAPERS

OF

EXAMPLES FOR PRACTICE.

N.B.—The Examples in the first twenty Papers are so arranged as to form Progressive Exercises in the subjects treated of in the Arithmetic.

PAPER I.

(1) i. Which is the greatest of the fractions $\frac{2}{3}$, $\frac{15}{18}$, $\frac{17}{24}$, $\frac{3}{4}$?

ii. Multiply the sum of the two first by the difference of the two last.

iii. Reduce them to decimals, and divide the continued product of the first three by the last.

(2) i. Find the value of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}$ of 16*l.* 8*s.*

ii. Add this result to $\dfrac{2\frac{1}{2}+7}{3+\frac{1}{2}}$ of $\dfrac{2\frac{2}{5}}{3\frac{4}{5}}$ of 2 guineas.

iii. Reduce i. and ii. to decimal fractions of 1*l.*, and divide their sum by ·125.

(3) i. Find the value of $1-3\frac{1}{2}+\frac{2}{3}+2\frac{4}{5}-\frac{1}{2}+\frac{4}{5}$.

ii. Multiply this result by such a fraction as will make it equal to 2.

iii. Reduce (i.) to a decimal and divide it by $5{\cdot}5-3{\cdot}75$.

(4) i. Find the value of $(2\frac{2}{3}-1\frac{1}{4})\left(3-\dfrac{4\frac{1}{2}}{3}\right)+2-\left(1-\dfrac{4\frac{1}{3}}{5}\right)\div\frac{2}{15}$.

ii. How many times does 18·75*l.* contain $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{9}{10}$ of 15s.?

iii. Subtract 2 hundredths 4 ten thousandths from 1 tenth 2 millionths.

(5) i. Find the value of 32 cwt. 3 qrs. 15 lbs. at 2*l.* 4*s.* 4*d.* per cwt. by practice.

ii. Find the value of 9 tons 2 cwt. 17 lbs. at 11*l.* 13*s.* 4*d.* per ton by practice.

iii. Find the value of 22 acres 3 roods 25 poles at 2*l.* 6*s.* 8*d.* per acre by practice.

Prove i. ii. iii. by Vulgar Fractions, Decimals, or Proportion.

(6) i. What is the net weight of 25 bags of cotton, each weighing 2 cwt. 3 qrs., tare 4 lbs. per 132 lbs., draft 2 lbs. in 500 lbs.?

ii. What is the net value of the above at 1*s.* 3*d.* per lb. after allowing $\frac{1}{8}$ per cent. for brokerage?

iii. A pipe of wine of 90 gallons pays an export duty at Bordeaux of 2 f. per hectolitre of $22\frac{1}{2}$ gallons; it costs 2*s.* 6*d.* a gallon to clear it in England, and $1\frac{1}{2}$ per cent. for expenses of cartage, &c.; the wine merchant's price was 1440 f. What did it cost altogether, and what is that per dozen bottles imperial (6 bottles imperial to the gallon)?

(7) i. What is the interest of 378*l.* 4*s.* $7\frac{1}{2}d.$ for 2 yrs. $28\frac{1}{3}$ days at 3*l.* 2*s.* 6*d.* per cent. per annum?

ii. What sum will amount to 376*l.* 12*s.* 11*d.* in 3 yrs. 2 months at $3\frac{3}{4}$ per cent.?

iii. What is the amount of $50\frac{2}{3}l.$ at $2\frac{1}{4}$ per cent. per annum for 75 days?

(8) i. Find the discount on a bill of 42*l.* 12*s.* 1*d.* payable in 9 months at 3 per cent. per annum.

ii. Find the difference between the bankers' and true discount on a bill of 1000*l.* payable at sight at 4 per cent.

iii. What is the present worth of each of the above bills?

(9) i. Dutch Rhenish Railway Stock was bought at a discount of 5 per cent., and 570*l.* was invested; how many 20*l.* shares were bought?

ii. What would be the half-yearly income in the last at 17*s.* 6*d.* per share per annum, deducting income-tax at 6*d.* in the pound?

iii. Transferred the above stock into London and North Western at 114; how much of the latter shall I hold?

(10) i. Bought 190 yds. of cloth at *5s. 6d.* per yd. and sold it for *7s. 8d.* per yd.; what did I gain?

ii. How much per cent. did I gain, and what must I have sold it for to realise 10 per cent.?

iii. And what must I ask to clear 10 per cent., after deducting 2 per cent. discount?

(11) i. A gives B 24 cwt. 2 qrs. 6 lbs. of cheese at *7d.* per lb. for cloth at 5s. per yd.; how many yards were there?

ii. If B had paid 10*l.* in cash, how many yards were there?

iii. Suppose A's cheese could be sold for ready money at *6d.* per lb., what would have been the ready money price per yard of the cloth?

(12) i. A, B, C, rent a field containing 20 acres 2 roods; A puts in 10 cows for 3 months, B 20 cows for 2 months, C 30 cows for $1\frac{1}{2}$ month; A pays *5l. 15s.* of the rent; what did B and C pay?

ii. What was the rent of the field per acre?

iii. If A puts in horses, B sheep, C cows, and 4 sheep eat as much as a cow, then how many horses will A put in if 5 cows are equivalent to 3 horses?

(13) i. A debt of 500*l.* is paid as follows: 200*l.* in three months, and the rest in monthly payments of 50*l.* each; when should the whole be paid at once?

ii. If the first payment bore interest at 5 per cent. per annum, and the other payments 4 per cent. per annum, when should the whole be paid, supposing interest for the whole payment be at 4 per cent. per annum?

iii. A debt of 900*l.* is paid as follows: $\frac{1}{2}$ down; of the remainder, $\frac{1}{3}$ in 3 months, $\frac{1}{6}$ in 6 months, $\frac{1}{9}$ in 8 months, and the rest in 10 months; when should the whole be paid at once?

(14) i. Find the square root of 2401686049; $3\frac{1}{16}$; 3·0276.

ii. Find the square root of ·000139; 25·250625.

iii. Find the cube root of 521660125; $4\frac{17}{27}$; 1006·012008.

(15) i. Exchange 2460 dollars for francs at 5·4 francs per dollar.

ii. Also supposing French money at $2\frac{1}{2}$ per cent. premium.

iii. Suppose a bill of 2460 dollars paid to a banker at Paris, and then to a merchant at Amsterdam, and by him to a house in London, French money being at a discount of 10 per mille at Amsterdam, and Dutch money at par in London; how much English money would it realise finally?

PAPER II.

(1) i. Reduce $\frac{5074}{159229}$ to its lowest terms.

ii. Add together $4\frac{1}{3}$, $\frac{3}{8}$ of $2\frac{1}{2}$, $\frac{1}{9}$ of $5\frac{1}{2}$, $7\frac{5}{12}$, $\frac{1}{24}$ of 89, and $\frac{1}{8}$.

iii. Simplify $\frac{7}{12}+13\frac{7}{10}-\frac{5}{8}$ of $3-\frac{7}{15}-2\frac{7}{16}+10\frac{1}{2}-\frac{7}{18}$.

(2) i. Find the continued product of $5\frac{5}{8}$, $\frac{3}{10}$ of $\frac{11}{18}$ of $3\frac{3}{7}$, $\frac{13}{14}$ of $9\frac{1}{3}$, $\frac{10}{11}$, and $7\frac{1}{3}$ of $\frac{1}{10}$, and reduce the result to a decimal.

ii. Which is greater, $\frac{1}{6}$ of $\frac{16}{27}$ of 12 of $\frac{5}{8}$, or $\frac{25}{32}$ of $\frac{4}{5}$ of $\frac{13}{15}$, and by how much? Reduce the difference to a decimal.

iii. Find the sum, difference, quotient, and product of $\frac{21}{44}$ and $\frac{7}{33}$.

(3) i. Express in a decimal and also in a fractional form the result of 34 hundredths + 2085 tenths + 17 thousandths + 59 + 375 millionths.

ii. 7 ninths + 11 hundredths + 120 tenths + 5 eighteenths + 13 twenty-sevenths + 4 thousandths.

iii. 219283 thousandths + 714 tenths − 162598 ten-thousandths − 3037 ten-thousandths − 2015 tenths + 7368 hundredths + 1809 tenths.

(4) i. What decimal multiplied by 562·5 will produce 37·345?

ii. What vulgar fraction multiplied by 562·5 will produce $37\frac{69}{200}$?

iii. Express as a vulgar fraction $19{\cdot}39\dot{2}8571\dot{4}$.

(5) i. Find the value of 635 tons 3 qrs. 18 lbs. at 3*l.* 14*s.* 8*d.* per cwt.

ii. Find the value of 15 cwt. 2 qrs. 22 lbs. at 53*l.* 10*s.* per ton.

iii. What must be given for $7395\frac{11}{12}$ cwt. at 12*l.* 19*s.* $10\frac{1}{2}$*d.* per cwt.?

(6) i. Find the net weight of 70 casks weighing 624 cwt., allow-

ing 7 lbs. per cwt. for the weight of the cask, tret 4 lbs. in 104 lbs., and draft 2 lbs. in 3 cwt.

ii. Suppose the casks in the preceding question to contain oil, how many gallons would they hold, allowing $12\frac{1}{2}$ lbs. to the gallon, and what would be the value at $11\frac{1}{4}$*d.* per quart?

iii. Find the net value of 130 boxes, each 2 cwt. 3 qrs. 14 lbs., tare 16 lbs. per box, and tret 4 lbs. in 104 lbs. at 65*l.* 6*s.* 8*d.* per ton.

(7) i. What will 927*l.* 10*s.* amount to at compound interest in 4 yrs. at $3\frac{1}{3}$ per cent.?

ii. What will 795*l.* 11*s.* 8*d.* amount to at simple interest in 3 yrs. 105 days at $9\frac{1}{8}$ per cent.?

iii. What sum will amount to 698*l.* 14*s.* $0\frac{3}{4}$*d.* in 7 yrs. 8 months at 7 per cent.?

(8) i. Find the true discount of 1021*l.* 18*s.* $5\frac{1}{4}$*d.* due 11 months hence at $5\frac{1}{4}$ per cent.; also the present worth of the same.

ii. Find the difference between the bankers' and true discount of 300*l.* 2*s.* 6*d.* for 4 months at $6\frac{1}{4}$ per cent.

iii. What will a banker gain by discounting a bill of 1333*l.* 6*s.* 8*d.* drawn on March 2nd at 8 months, and discounted on the 8th of June at $6\frac{1}{4}$ per cent.?

(9) i. How much Danish $4\frac{1}{2}$ per cent. Stock at $94\frac{3}{4}$ can be bought for 3790*l.*?

ii. What will be the yearly income from this investment?

iii. If a yearly income of 17*l.* 11*s.* is acquired from the above stock after deducting income tax at 6*d.* in the 1*l.*, what capital was invested?

(10) i. If 60 yds. of cloth cost 20*l.*, how must I sell it per yard to clear 1*l.*?

ii. How much per cent. shall I gain on the prime cost?

iii. Should I gain or lose by selling 20 yds. of the above at 6*s.* 6*d.* per yard, and the rest at the price in (i.), and how much per cent. on the cost?

(11) i. How much beef at 10*s.* 8*d.* a stone of 16 lbs. is worth 26 yds. of linen at 2*s.* 7*d.* per yard?

ii. How much sugar at 40*s.* per cwt., together with 18*l.* in cash, must A give B for 180 yds. of linen at 4*s.* 6*d.* per yard?

iii. How many yards at 3*s.* a yard, together with 210 yds. at 5*s.* 6*d.* per yard, and 10*l.* in cash, must A give B for 6 chests of tea, each weighing 80 lbs. at 3*s.* per lb.?

(12) i. A, B and C enter into partnership; A with a capital of 100*l.*, B and C each with 250*l.*, and they gain $\frac{1}{20}$ part of their whole capital; find A's share.

ii. If A had entered with 100*l.*, and B and C had shared 15*l.* and 25*l.* respectively out of a profit of 68*l.*, how much capital did they advance?

iii. If A had put in 100*l.* for 5 years, and B 250*l.* for 3 years, for how long must C have entered his capital to have shared half as much as A and B together?

(13) i. If 40*l.* be due in 9 months, and 60*l.* in 4 months, when should the whole be paid at once?

ii. If 100*l.* is due in 4 months, 200*l.* in 5 months, and 600*l.* in 12 months, find the equated time for paying the whole.

iii. Suppose in (ii.) the interest of the first 100*l.* had been 2 per cent., of the 200*l.*, 3 per cent., and of the 600*l.*, 4 per cent., find the equated time, reckoning the interest on the whole debt at 5 per cent. per annum.

(14) i. Find the square root of ·000040422943273216.

ii. Find the square root of 7 to eight figures.

iii. Find the cube root of 59730·618429.

(15) i. Exchange 850*l.* for francs at 25·15 francs per 1*l.*

ii. If English money be at 5 per cent. premium at Paris, find the exchange.

iii. If English money be at a discount of 10 per cent. at Paris, find the exchange.

PAPER III.

(1) i. What is the least number of penny pieces that can be piled in columns of 7, 9, 10, 11, 12, 13, 14, 18, 20, 22 or 26, so that none remain?

ii. Four persons, A, B, C, D, have respectively 9464, 21112, 3848 and 1144 sovereigns, and each divides his money into an exact number of heaps of the same value; what is the greatest number of sovereigns that each heap could possibly contain, and how many heaps would each person then have?

iii. Reduce to their simplest form $\frac{72369}{614771}$; $\frac{5382}{38493}$.

(2) i. Find the continued product of $7\frac{1}{2}$, $\frac{2}{3}$ of $\frac{11}{15}$, $3\frac{7}{10}$, $4\frac{3}{4}$, $\frac{17}{74}$ of $\frac{16}{51}$ of 23; also give the result in a decimal form.

ii. Express $\frac{3}{5}$ of $\frac{7}{8}$ of $\frac{15}{21}$ of 1*l.* 17*s.* 6*d.* as a decimal of $4\frac{1}{6}$*l.*

iii. Bought $\frac{3}{4}$ of $\frac{5}{6}$ of an estate worth 37,525*l.* and sold $\frac{7}{10}$ of my share; what portion had I left, what did I sell? and find the value of each.

(3) i. Which is the greatest and which the least of the following fractions? Find the difference between the greatest and least, and reduce it to a decimal: $\frac{5}{12}$, $\frac{2}{3}$, $\frac{7}{16}$, $\frac{19}{24}$, $\frac{4}{5}$, $\frac{15}{32}$.

ii. Simplify $\dfrac{11\frac{1}{5} \text{ of } 1\frac{5}{8} \text{ of } \frac{20}{39}}{\frac{2}{7} \text{ of } \frac{3}{4}} + \dfrac{1\frac{1}{3} \text{ of } 7}{\frac{11}{12} \text{ of } 4\frac{4}{11} \text{ of } \frac{7}{10}}$.

iii. Add together $7\frac{1}{3}$, $2\frac{3}{4}$, $\frac{4}{7}$ of $\frac{2}{3}$, and 13; from this sum subtract $1\frac{11}{14} + 2\frac{5}{7} + 2\frac{15}{28}$, and divide the result by $\frac{4}{5}$ of $\frac{11}{36}$, expressing the answer in a decimal form.

(4) i. Find the continued product of $4{\cdot}\dot{0}\dot{9}$, ${\cdot}91\dot{6}$ and ${\cdot}0125$.

ii. Add together $19{\cdot}375$, ${\cdot}0\dot{6}\dot{3}$, $11{\cdot}8$, $9{\cdot}\dot{6}$ and ${\cdot}375$.

iii. Express as a decimal $\dfrac{5\frac{1}{9} \text{ of } \frac{18}{23} \text{ of } 3 \text{ of } \frac{11}{18}}{(\frac{3}{4} \text{ of } 16) + (\frac{1}{3} \text{ of } 4\frac{1}{2})}$

(5) i. Find the dividend on 4106*l.* 12*s.* 6*d.* at 17*s.* $7\frac{1}{2}$*d.* in the pound.

ii. What will a creditor lose whose claim is 6741*l.* 15*s.* 0*d.* if he only receives 17*s.* $10\frac{1}{4}$*d.* in the pound?

iii. Find the value of 57 chests of tea, each 3 qrs. $17\frac{1}{2}$ lbs., at 3*s.* $10\frac{1}{2}$*d.* per lb.

(6) i. What is the net weight of 100 barrels of flour weighing 219 cwt. 1 qr. 14 lbs., tare 12$\frac{4}{9}$ lbs. per cwt., tret 4 lbs. in 104 lbs., and draft 2 lbs. in 3 cwt.?

ii. 525 boxes of butter weighed gross 616 cwt. 1 qr. 7 lbs.; find the net weight after allowing 15 lbs. for the weight of each box, tret 4 lbs. in 104 lbs., and draft 2 lbs. in 3 cwt.; what would it amount to at 1*s.* 3$\frac{1}{4}$*d.* per lb.?

iii. Find the value of 180 casks of turpentine weighing 750 cwt. 1 qr. 18 lbs., at 7*s.* 2$\frac{1}{4}$*d.* per gallon, allowing 48 lbs. for the weight of each barrel and 7$\frac{1}{2}$ lbs. to a gallon.

(7) i. At what rate per cent. will the interest on 918*l.* 16*s.* 8*d.* amount to 166*l.* 10*s.* 9$\frac{1}{4}$*d.* in 2 yrs. 5 months?

ii. What sum will 1180*l.* 12*s.* 6*d.* amount to at simple interest in 10 yrs. 292 days at 9$\frac{4}{9}$ per cent.?

iii. What sum lent at compound interest will amount to 303*l.* 17*s.* 6$\frac{3}{8}$*d.* in 4 years at 5 per cent.?

(8) i. Find the true discount of 1021*l.* 18*s.* 5$\frac{1}{4}$*d.* for 11 months at 5$\frac{1}{4}$ per cent.

ii. A bill of 341*l.* 5*s.* was drawn on Dec. 22 at 8 months, and discounted April 29 at 7$\frac{1}{2}$ per cent.; what sum will the holder of the bill receive (bankers' discount)?

iii. Find the true discount in the preceding question.

(9) i. What will be the purchase of 53*l.* 5*s.* stock at 85$\frac{1}{3}$?

ii. What income will arise from transferring it to the 5 per cent. Stock at 105?

iii. The half-yearly dividend of a certain sum invested in 4$\frac{1}{4}$ per cent. Stock is 17*l.* If the Stock was bought at 90$\frac{1}{8}$ per cent. including the broker's commission of $\frac{1}{8}$ per cent., how much money was invested?

(10) i. Bought 6 chests of tea each 88 lbs. weight for 66*l.*, and sold it at a profit of 26*l.* 8*s.*, what was it sold for per lb.?

ii. If I give 2*s.* 6*d.* for an article that I sell for 3*s.* 4*d.*, what do I gain per cent.?

iii. If I sold an article for 3*s.* 2*d.* and lost 5 per cent., what did it cost?

(11) i. What is cloth a yard if I receive 16 yds. $2\frac{6}{25}$ qrs. for 7 cwt. 2 qrs. of sugar at 1*l.* 12*s.* per cwt.?

ii. How much cheese at 30*s.* per cwt. must A give B for 10*l.* in cash, and 30 yds. of cloth at 3*s.* 4*d.* per yard?

iii. If A charged B 30*s.* in barter for what he sold at 27*s.* ready money, what would have been the ready money price of B's cloth?

(12) i. A, B and C purchase an estate of 78 acres of land and a house, the whole is sold for 9000*l.* If A and B take all the land but 4 acres, which C takes with the house, what must each pay if the house be worth 12 acres of land?

ii. A and B engage a room for a concert, and they are to pay $\frac{1}{3}$ of the proceeds. The concert realised 250*l.* from tickets, the expenses were $\frac{3}{5}$ of the receipts, and B was to have $\frac{1}{8}$ more than A of the surplus; what did he receive?

iii. A, B, C, engage a field for 45*l.* A puts in 200 sheep, B 50 sheep for 5 months, and C pays 20*l.*; how many sheep did he put in for 9 months, and for how long a time did A use the field?

(13) i. A owes B 240*l.* to be paid in 6 months, but in $1\frac{1}{2}$ months he pays him 60*l.*, and in $4\frac{1}{2}$ months 80*l.* more; when should he pay the rest?

ii. If in (i.) A had paid 100*l.* down, when should he pay the remainder at once?

iii. 200*l.* is due in 12 months; if $\frac{1}{2}$ is paid in 4 months, $\frac{1}{4}$ in 6 months, and $\frac{1}{5}$ in 9 months, to what time could the remainder be deferred?

(14) i. Extract the Square Root of $\frac{58081}{62500}$ in a fractional and also in a decimal form.

ii. A nobleman purchased an estate containing 86051 a. 2 r. 20 p. 3 ft. 9 in. of land; if it were a perfect square how many miles long would each side be?

iii. A cube contains $2326\frac{13}{64}$ inches; what is the length of its side?

(15) i. Exchange 825*l.* for marks at 13 marks 12 sch. per 1*l.*

ii. Suppose in (i.) English money at a premium of 10 per cent. at Hamburg?

iii. Suppose in (i.) English money at a discount of 10 per cent. at Hamburg?

PAPER IV.

(1) i. Express 2*l.* 3*s.* $6\frac{1}{4}$*d.* as a Vulgar Fraction and also as a decimal of 5*l.*

ii. Express in their simplest form $\dfrac{4147}{20793}$; $\dfrac{16 \times \frac{9}{32} \text{ of } \frac{1}{324}}{\frac{5}{7} \text{ of } 12\frac{3}{5} \times \frac{4}{9}}$.

iii. Add together $3\frac{17}{32}$ of a sovereign, $\frac{19}{81}$ of a moidore, $1\frac{5}{8}$ of a guinea, and $\frac{7}{18}$ of 10*s.* 6*d.*

(2) i. What number must the numerator and denominator of the fraction $\frac{37}{318}$ be multiplied by in order to make it equal to $\frac{2923}{24964}$? and reduce $\frac{37}{318}$ to a decimal.

ii. Express 7 furlongs 20 poles 5 yds. 2 ft. 1 in. as a decimal of 1 mile 1 fur. 1 pole 1 yd. 1 ft.

iii. Divide $\dfrac{\frac{7}{8}}{3\frac{1}{9}}$ of $\dfrac{1\frac{4}{9}}{\frac{13}{36}}$ sovereigns among 8 poor persons; what will each receive?

(3) i. From the sum of 5845 hundredths, 392 tenths, and one thousand two hundred, take the sum of 13·21693, 75016 thousandths, 385·2398, ·206 and fifteen.

ii. Find the continued product of 4675 hundredths, 467 millionths, and 946 thousandths.

iii. Simplify $\dfrac{20{\cdot}125}{{\cdot}7291\dot{6}}$; $\dfrac{{\cdot}\dot{5}3846\dot{1}}{\frac{7}{26} \text{ of } 8}$

(4) i. Add together $16{\cdot}\dot{3}$, $12{\cdot}0\dot{3}$, ${\cdot}0\dot{6}$, $10{\cdot}\dot{8}5714\dot{2}$ and $11\frac{1}{3}$.

ii. Divide the product of $17{\cdot}\dot{1}$ and ${\cdot}10\dot{7}1428\dot{5}$ by $\dfrac{1}{1{\cdot}6\dot{3}}$ of 4.

iii. A gentleman left 9600*l.* to be divided as follows; $\frac{1}{3}$ to A, $\frac{1}{8}$ of the remainder to B, $\frac{1}{7}$ of what was then left to C; to D half the share of C, to E 100*l.* more than to A, and the rest equally among F, G and H; find the share of each.

(5) i. Find the cost of 3 bags of coffee each 5 cwt. 1 qr. 11 lbs. at 1*s.* $11\frac{1}{4}$*d.* per lb.

ii. A gentleman being asked how much he was worth, replied: 'For every square inch in my estate, which has an area of 57 acres 2 roods 36 poles 3 ft. 27 in., I am worth a farthing.' Calculate this.

iii. Find the value of 4 tons 17 cwt. 3 qrs. 23 lbs. at 1*l.* 3*s.* 4*d.* per cwt.

(6) i. Find the net value of 65 bags of cinnamon, each 3 cwt. 1 qr. 5 lbs., allowing $11\frac{1}{2}$ lb. for the weight of each bag, at $1\frac{1}{2}$*d.* per oz.

ii. What is the net weight of 36 casks of sugar, weighing gross 221 cwt. 3 qrs. 14 lbs., tare 18 lbs. per 100 lbs., and tret 4 lbs. in 104 lbs.?

iii. What is the net weight of 8 bales of cotton, each 5 cwt. 2 qrs. 16 lbs., tare $24\frac{1}{2}$ lbs. per bale, and tret 28 lbs. in 108 lbs.?

(7) i. What sum lent at simple interest at $10\frac{2}{7}$ per cent. for $19\frac{4}{9}$ years will amount to 825*l.* 10*s.* $9\frac{3}{4}$*d.*?

ii. In what time will 228*l.* 8*s.* 9*d.* amount to 303*l.* 16*s.* $5\frac{1}{4}$*d.* at 6 per cent.?

iii. What will 725*l.* 6*s.* 8*d.* lent at simple interest at 5 per cent. from March 13th, 1847, to April 1st, 1848, amount to?

(8) i. Find the present worth of 1141*l.* 8*s.* 9*d.* for 1 yr. 7 months at $5\frac{1}{2}$ per cent.

ii. What would be the present worth in the preceding question, allowing bankers' discount?

iii. What will a banker gain by discounting a bill of 365*l.* 10*s.* 3*d.* due 15 months hence at $5\frac{1}{2}$ per cent.?

(9) i. Bought Stock at $92\frac{1}{2}$, sold it at 95, and realised 380*l.*; how much stock did I buy, and how much did I gain?

ii. Transferred 500*l.* from the 4 per cents. at 84, to the 3 per cents. at 66; what was the change in income?

iii. How much of the 3 per cent. Stock in (ii.) must I have to get the same income as I did from the 4 per cent. Stock?

(10) i. Bought sugar at 42*s.* per cwt., kept it in warehouse 3 months at a charge of 2*d.* per month per cwt., and sold it to gain 10 per cent. on the whole cost; what did I sell it for per cwt.?

ii. Bought 6 chests of tea for mixing; the prices per lb.

were 2*s*., 2*s*. 2*d*., 2*s*. 4*d*. for 3 chests, and 3*s*. for the remainder. Two lbs. of each of the first three are mixed with $1\frac{1}{2}$ lbs. from each of the others. If I gain 5*l*. per cent. on the cost, what do I sell the mixture at per lb.?

iii. If I had sold the mixture at 3*s*. per lb., and gained nothing, how many lbs. would there have been in each chest?

(11) i. How many yards of linen at 4*s*. 1*d*. per yd. must be given for 125 yds. of cloth at 17*s*. $4\frac{1}{2}$*d*. per yd.?

ii. Bought 2 hhds. of sugar weighing gross 15 cwt. 2 qrs. each at 36*s*. per cwt. net; paid $\frac{3}{4}$ in cash, and the remainder in linen at 4*s*. 4*d*. per yd.; how much money, and how much linen did I give, tare allowed at 14 lbs. per hogshead?

iii. Bartered coffee at 8*d*. per lb. profit, for tea at 6*s*. 4*d*. per lb. which cost 5*s*. per lb.; what did the coffee cost per lb.?

(12) i. A, B, C speculate and gain $\frac{4}{15}$ of their capital, and they share the profits thus: $\frac{2}{3}$ to A, $\frac{1}{4}$ to B, and the rest to C; what was the proportion of A's capital to B's?

ii. A and B hire a vessel. A's freight consists of 2000 yds. of calico at $10\frac{1}{2}$*d*. per yd., 200 dozens of handkerchiefs at $7\frac{3}{4}$*d*. each, and 40 bales of stuffs worth twice as much as his other goods; B's freight was wine and beer to the amount of 500*l*. They insure at 3 per cent., and the whole expense is 150*l*.; what does A pay towards the hire of the vessel?

iii. 10 men, 25 boys, and 30 women are employed in agriculture. A man has 2*s*. 3*d*. per day, a boy $\frac{2}{3}$ of what a woman has, and a woman has $\frac{3}{16}$ of the wages of 4 men. In what time will they earn 81*l*. altogether?

(13) i. A owes B three bills of 20*l*. 6*s*. 8*d*., 90*l*. 2*s*. 6*d*. and 105*l*. 5*s*.; they are due in 3, 4, 8 months respectively, but he pays them all in 5 months. Does he lose or gain by this, and how much (interest at 5 per cent.)?

ii. A has to pay 250*l*. by instalments of 50*l*. at the end of every 2 months. He pays the first two regularly; when should he pay the remainder at once to clear 1*l.* discount, interest at 5 per cent.?

iii. 900*l.* is borrowed on mortgage thus: 4 per cent. per annum is the rate of interest, and the loan is to be paid in 30 years (beginning with the 2nd year) by instalments of 30*l.*; what will be the sum to be paid, of capital and interest, on the tenth payment?

(14) i. Extract the square root of $\frac{37}{2560}$ to 6 decimal places, and prove it.

ii. Extract the cube root of $\frac{23}{175}$ to 3 decimal places, and prove it.

iii. The area of a square field is 328134·2089 inches; find the length of its side in yards.

(15) i. Exchange 500*l.* for rupees (1*s.* $11\frac{3}{4}$*d.* = 1 rupee).

ii. If the exchange was 1*l.* = $23\frac{1}{2}$ rupees, at what discount would gold be in India?

iii. Exchange 3764 rupees for sterling money at $28\frac{3}{4}$*d.* per rupee.

PAPER V.

(1) i. Simplify $3\frac{2}{7} + \frac{11}{14} + 5\frac{3}{8} + \frac{17}{21} + \frac{1}{28} + 3\frac{22}{35}$.

ii. Simplify $3\frac{7}{11} - 2\frac{4}{9} + \frac{13}{22} - \frac{25}{144} + 14\frac{1}{7} + 2\frac{61}{63}$.

iii. Simplify $\dfrac{\frac{19}{27} - \frac{25}{108}}{\frac{4}{9} \text{ of } \frac{11}{16}} - \dfrac{\frac{11}{12} - \frac{13}{84}}{\frac{9}{14} + \frac{5}{42}}$.

(2) i. Express as mixed numbers $\frac{4968358}{533}$; $\frac{78426}{312}$; $\frac{7346375}{625}$.

ii. Find the lowest terms of $\frac{4144}{15568}$; $\frac{3240}{20304}$; $\frac{6103}{127804}$.

iii. Express as improper fractions $1869\frac{535}{7189}$; $3479\frac{51}{497}$; $818\frac{641}{956}$.

(3) i. A man bequeathed his estate worth 35,240*l.* to his four sons, in the following manner: to the eldest $\frac{1}{3} + \frac{1}{15}$ of the whole; to the second $\frac{5}{12} - \frac{1}{6}$; to the third ($\frac{2}{9}$ of $\frac{9}{20}$) + $\frac{1}{2}$; and the remainder to the youngest. Find the share of each; and what fraction of the second son's share is that of the youngest?

ii. What decimal of 15 crowns is $\frac{1}{8}$ of ·125 of 2 florins?

iii. What decimal is equivalent to $\dfrac{\frac{16}{81} \text{ of } \frac{1}{4} \text{ of } \frac{5}{63}}{3\frac{5}{9} \text{ of } \frac{5}{7} \text{ of } \frac{4}{9}} + \dfrac{2\frac{5}{9} + \frac{5}{12} - \frac{13}{16}}{\frac{7}{15} \text{ of } \frac{1}{20} \times \frac{25}{84}}$

(4) i. Express $2{\cdot}\dot{6} + 17{\cdot}1\dot{6} + {\cdot}41\dot{6}\dot{6} + 22{\cdot}625$ as an equivalent vulgar fraction.

ii. Express in positive terms ·00325 of ½ sovereign + 1·075 of a crown + 13·25 of a florin.

iii. From 17·075 of 5 guineas take 2·385 of *5l.*, and from the remainder deduct ·00625 of 10*s.* 6*d.* Express the result in positive terms.

(5) i. Bought a hhd. of sugar weighing 13 cwt. 1 qr. 14 lbs. at 8¼*d.* per lb., and also 2 others averaging 12 cwt. 2 qrs. 19 lbs. at 4 guineas per cwt., find the cost of the whole.

ii. Find the value of 37 ingots of gold, averaging 23 lbs. 2 oz. at 3¼*d.* per grain.

iii. How much must be given for a gold cup weighing 9 oz. 10 dwts. 18 grains at 40 guineas per lb.?

(6) i. Find the net weight of 16 hhds. of tobacco weighing 64 cwt. 2 qrs., tare 4 cwt. 2 qrs. 14 lbs., tret 4 lbs. in 104 lbs., and draft 1 lb. in 1½ cwt.

ii. Find the net value of 200 boxes of tallow, weighing 357 cwt. 1 qr. 16 lbs., allowing 46 cwt. 16 lbs. for the weight of the boxes, and tret 4 lbs. in 104 lbs., at 3¼*d.* per lb.

iii. How many boxes of raisins, each holding 25 lbs., can be filled out of 27 hhds. weighing 211 cwt. 1 qr., allowing $8\frac{8}{13}$ lbs. per cwt. for tare, tret 4 lbs. in 104 lbs., and draft 2 lbs. in 3 cwt.?

(7) i. What sum lent at compound interest for 3 yrs. at 7½ per cent. will amount to 1242*l.* 5*s.* 11¼*d.*?

ii. Find the simple interest of 927*l.* 10*s.* for 2 years 175 days at 2¾ per cent.

iii. What sum lent at simple interest for 4 years 73 days at 7⅕ per cent. will amount to 838*l.* 8*s.* 4⅘*d.*?

(8) i. Find the true discount on a bill of 110*l.* 7*s.* 1½*d.* for 7½ months at 6¼ per cent.

ii. A bill of 3200*l.* due 11 months hence at 4¾ per cent. is discounted what shall I receive from the banker?

iii. What ought I to receive, reckoning true discount for a bill of 135*l.* 13*s.* 2½*d.* due 11 months hence at 4¾ per cent.?

(9) i. A invests 500*l.* in the 4 per cents. at 90, and B 315*l.* in the 5 per cents. at 105; find the income of each.

ii. Suppose the 4 per cents. to rise to 92, and the 5 per cents. to fall to 100, and A to transfer his stock to the 5 per cents., and B his to the 4 per cents.; find the alteration in their incomes.

iii. If 20*l.* railway shares are sold at 16*l.*, and the dividend for the half year is declared at 3½ per cent. per annum, how much will a holder of 25 shares receive, after deducting income-tax at 6*d.* per 1*l.*, and what is the rate of interest on his investment?

(10) i. Sold an article for 4*s.* 0½*d.* and lost 3 per cent., what should I have gained by selling it for 4*s.* 4*d.*?

ii. Bought 10 cows and 16 sheep for 216*l.* If a cow cost 12½ per cent. more than a sheep (which cost 2*l.* 5*s.*), what must I sell the cows for to gain 10 per cent. upon the whole?

iii. What must I ask for an article to obtain 12*s.* 6*d.* for it, after deducting a discount of 10 per cent.?

(11) i. Sold 24 pieces of silk for 5 chests of tea, weighing net 80 lbs. per chest; the silk was 3*s.* 4*d.* per yard, and the tea was 5 per cent. dearer per lb. than the silk per yard; how much silk was there in each piece?

ii. Bartered cheese for tea in the proportion of 7 lbs. of cheese to 2 lbs. of tea; the value of the latter was 10*l.*, and there were 2 cwt. of cheese; how much tea was there, and what was the price of the tea and cheese per lb.?

iii. If the cheese in the last question had been 10 per cent. dearer, and the tea 12 per cent. cheaper, what would have been the proportion in barter?

(12) i. A bankrupt owes 2002*l.* to five creditors, whose accounts are proportional to the numbers 2, 3, 4, 5, 6, but his assets only amount to 210*l.*; how much is his largest debt, and also the smallest payment?

ii. A, B and C are in partnership for 20 years, and they began with capitals 100*l.*, 200*l.*, 300*l.* respectively. The first 5 years they gained 1000*l.*, in the next 5 they lost 500*l.*, and in the last 10 years they gained enough to retire upon incomes equal to their capitals at first invested. Now, supposing them to get 5 per cent. per annum for their money, what did each gain in the last 10 years?

iii. Suppose in the last question C had retired at the end of the tenth year, paying 50*l.* to the firm to be shared in proportion to capital, and B at the end of the fifteenth year, what would A have been worth at the end of the twentieth year, supposing the same circumstances of profit and loss, and each had capitalised his gain and proportion of C's. fine?

(13) i. A owes B 24 cwt. of rice, 10 chests of tea, each 80 lbs. at 2*s.* 6*d.* per lb. net, tare 8 lbs. per chest, and 20 cwt. of sugar at 3*d.* per lb. He pays for the rice by a bill at 3 months, and the price of it is half that of the sugar; the tea he pays for in 4 months, and the sugar in 5 months; when should he pay the whole at once to the same advantage?

ii. Out of 3 bills (200*l.* at 4 months, 100*l.* at 6 months, and 350*l.* at 3 months), the first and last are paid at the end of 2 months; to what time can the payment of the second be deferred?

iii. 270*l.* is to be paid in monthly payments of 30*l.* each; what advantage would arise from paying it in 3 half-yearly payments?

(14) i. Extract the square root of 1916·341 to 5 figures, and ·7 to 8 figures.

ii. Find the side of a square field equal in area to one of a rectangular shape whose length is 4800 ft. and breadth 1200 ft.; and how many steps would a person take in going once round, if he touched the ground 3 times every $7\frac{1}{2}$ feet.?

iii. A cubical mound contains $1911\frac{1401}{1600}$ inches, what is the length of its sides?

(15) i. A Chinese dollar or tael is = 4*s.* $7\frac{3}{8}$*d.* sterling. Exchange 500*l.* for taels at 55*d.*

ii. Exchange $3180\frac{4}{5}$ taels at $55\frac{3}{8}$*d.* for English money.

iii. If English money is at 5 per cent. premium in China, what will be the results in (i.) and (ii.) ?

PAPER VI.

(1) i. Find the G. C. M. of 5916, 94308 and 182787 ; and what is the greatest number that will divide 4929 and 43672 ?

ii. Find the L. C. M. of 3, 7, 11, 15, 21, 22, 28, 32 and 50 ; and what is the least number which is exactly divisible by 14, 15, 16, 18, 20 and 24 ?

iii. Express $\frac{431}{2308}$ as a fraction with 92320 for its denominator, and also $\frac{151}{2001}$ as a fraction with 96048 for its denominator.

Perform the following operations :—

(2) i. $4\frac{5}{8}$ of $12 \times \frac{13}{21}$ of $\dfrac{\frac{29}{111}}{\frac{11}{56}} \times \dfrac{4\frac{5}{7}}{1\frac{14}{15}}$; $\dfrac{5\frac{5}{8}}{1\frac{1}{4}}$ of $\frac{13}{14}$ of $\dfrac{\frac{9}{13}}{3\frac{5}{7}} \times \dfrac{7\frac{3}{7}}{\frac{1}{5}}$.

ii. $(4\frac{10}{11}$ of $3\frac{4}{5}) \div (7\frac{9}{13} \times 1\frac{13}{14}$ of $\frac{28}{33}$ of $\dfrac{2\frac{1}{7}}{\frac{2}{19}} \times \frac{19}{25})$; $\dfrac{\frac{1}{6} \times \frac{16}{27} \times \frac{25}{32} \times 5\frac{2}{15}}{1\frac{1}{4} \text{ of } \frac{11}{27} \text{ of } \frac{1}{12}}$

iii. $\dfrac{2\frac{17}{19} \text{ of } \frac{2}{5} \times 1\frac{1}{15}}{1\frac{13}{38}} \div \dfrac{3\frac{1}{4} \text{ of } \frac{5}{11}}{\frac{5}{12} + \frac{17}{24}}$.

(3) i. Express in positive terms 7·375 of a ton + ·625 of a cwt. + $31\frac{1}{4}$ qrs. + ·75 lb.

ii. What decimal of a league are 3·3 yards, and square the result ?

iii. Divide 27·9 by 1562·5 ; and 579·375 by 5·25.

(4) i. Find the continued product of 1234 thousandths, 1234 ten-thousandths, and ·01234.

ii. Among how many poor persons can 796*l.* 12*s.* $7\frac{3}{4}$*d.* be divided, giving to each $5{\cdot}6041\dot{6}$*s.* ?

iii. Express as vulgar fractions $10{\cdot}03\dot{3}7\dot{8}$; $\cdot842\dot{6}$; ·229376.

(5) i. Find the rent of a farm of 67 acres 2 roods 38 poles at 3*l.* 6*s.* 8*d.* per acre.

ii. The rental of a parish is 6116*l.* 16*s.* 8*d.*, how much will a rate of 3*s.* $11\frac{3}{4}$*d.* in the pound produce?

iii. If I pay 238*l.* 18*s.* 8*d.* per cwt., find the cost of 720 lbs. 10 oz. 14 drs.

(6) i. Find the net value of 87 cwt. 3 qrs. of tea, tare 17 lbs. per cwt., and tret 4 lbs. in 104 lbs., at $3\frac{1}{4}$*d.* per oz.

ii. Find the net weight of 37 barrels of apples, each 5 cwt. 3 qrs. 17 lbs., allowing 1 qr. 21 lbs. for the weight of the barrel. How many apples did they contain, supposing each weighed 4 oz.?

iii. Find the net weight of 725 barrels, weighing 1248 cwt., tare $17\frac{1}{2}$ lbs. per cwt., tret 4 lbs. in 104 lbs., and draft 2 lbs. in 3 cwt.

(7) i. What will 666*l.* 13*s.* 4*d.* lent at compound interest for $3\frac{1}{4}$ yrs. at $6\frac{1}{2}$ per cent. amount to?

ii. At what rate per cent. will the interest of 373*l.* 13*s.* 4*d.* amount to 138*l.* 14*s.* $5\frac{7}{10}$*d.* in 5 years 6 months?

iii. In what time will 679*l.* 13*s.* 9*d.* amount to 972*l.* 8*s.* $1\frac{1}{2}$*d.* at $7\frac{3}{8}$ per cent.?

(8) i. What is the present worth of 1249*l.* 7*s.* $8\frac{3}{4}$*d.* due 1 year $6\frac{1}{2}$ months hence, at $4\frac{1}{2}$ per cent.?

ii. Find the bankers' and true discount of 8963*l.* 15*s.* 10*d.* due in 75 days at $4\frac{1}{2}$ per cent.

iii. Find the present worth of a bill of 701*l.* 4*s.* $2\frac{3}{4}$*d.* due 4 months hence, and also find the interest of the present worth for the same time at $3\frac{1}{2}$ per cent.

(9) i. A person bought 5000*l.* Stock at 2 per cent. premium, also 2000*l.* Stock at 20 per cent. discount; how much 4 per cent. Stock at 95 will he get by selling out the first Stock at par and the second Stock at 78?

ii. What will be the broker's charge for all the transactions in (i.) at $\frac{1}{8}$ per cent. for commission and $\frac{1}{4}$ per cent. for incidental expenses and duty?

N.B. Transferring Stock from one kind to another is one transaction.

iii. What rate of interest arises from investing in the 4 per cents. at 84? And whether would this or the 5 per cents. at 95 be the better investment?

(10) i. Bought 15 lbs. of tea for a certain sum and sold 10 lbs. of it for the cost of the whole, find the gain per cent.

ii. At what price per lb., compared with the cost, should he have sold the remainder to clear 20 per cent. upon the whole?

iii. Bought an article for 16*s.* 7½*d.*, what must it be sold for to clear 12½ per cent. profit, after allowing 5 per cent. discount?

(11) i. A has 96 bushels of wheat, and B has 10 sacks of peas each 3 bushels. If wheat is worth 2 per cent. more than peas, and wheat is 8*s.* per bushel, how much wheat will A have left, after barter for the peas?

ii. If the peas had been 4 per cent. dearer, how much cash must B have given to A, so as to have taken all the wheat?

iii. A had a cargo of rice, and the expense of freight was 10 per cent. of its value. He had borrowed 5000*l.* upon it at 6 per cent. per annum interest; he kept the rice for 75 days and then exchanged it, with all these liabilities, for a cargo of maize worth half as much again as the rice, which was 12*s.* 6*d.* per cwt.; what was the quantity of maize?

(12) i. A and B hire a field for 15 months. A puts in 10 horses, 12 cows, and 50 sheep; how many cows can B keep for the time, and pay half the rent, if 2 cows or 5 sheep are equivalent to a horse?

ii. Suppose the rent is 63*l.* 6*s.* 8*d.*, how many cows could B keep for 6 months, and pay half the rent, if A's cattle remained the same, and the time was 15 months?

iii. Suppose in (i.) A had sold his horses at the end of 4 months, and his sheep at the end of 12 months, and B had bought them and brought them into pasture with his first stock until the end of the time, what would have been their shares of the rent?

(13) i. A Bill of 200*l.* is paid by A in 4 half-yearly payments of

50*l.* each, find when the first three may be paid at once.

ii. Suppose as in (i.) that each of the payments after the first bears interest at 3 per cent. until the time of payment, when could the whole be paid at once if he could get 4 per cent. per annum for his money?

iii. The Governors of Queen Anne's Bounty advance to A 900*l.* on mortgage of a living, and the loan is to be paid by 30 instalments of 30*l.* each, beginning at the second year, and interest at 4 per cent. per annum from the commencement of the loan; what would be gained or lost by A if he found the money and dispensed with the loan, reckoning interest of his money at $4\frac{1}{2}$ per cent. per annum?

(14) i. Find the square root of 5 to six figures, and of ·017 to eight figures.

ii. Find the cube root of 127268840·262941343.

iii. A cubical mound of earth contains 8908·491937491901 ft., find the area of the ground which it covers.

(15) i. Exchange 11363 francs, 25 cents. at Paris with Rotterdam for florins ($208\frac{1}{2}$ francs = 100 florins), and thence with Hamburg at $187\frac{1}{2}$ francs for 100 marks.

ii. Suppose in (i.) French money had been at a premium of $\frac{1}{2}$ per cent., what would have been the number of florins?

iii. A Bill on Amsterdam is bought in London at 12 florins 15 cents. per 1*l.*, and is sold in Paris at 57·50 florins per 120 francs, what is the exchange between London and Paris?

PAPER VII.

(1) i. A man bequeathed $7\frac{5}{11}$ of $\frac{13}{41}$ of $\frac{19}{20}$ of $\dfrac{4\frac{4}{19}}{3\frac{1}{17}}$ of $5\frac{1}{2}$ of 20 guineas to his relations in the following manner: $\frac{1}{12}$ to each of his four nephews, $\frac{1}{7}$ of the remainder to each of his five nieces, and the rest to be divided equally

among his four cousins; find the share of each, and what was the whole amount bequeathed?

ii. From $5\frac{5}{14}$ cwt. + $7\frac{11}{28}$ tons + $3\frac{7}{8}$ lbs., take $\frac{17}{32}$ cwt. + $1\frac{5}{64}$ qr. + $\frac{17}{32}$ ton.

iii. Which is the greatest of the fractions $\frac{3}{5}$, $\frac{7}{10}$, $\frac{9}{16}$, $\frac{5}{8}$, $\frac{17}{20}$, $\frac{19}{30}$, $\frac{29}{32}$? Find the sum, difference, product and quotient of the greatest and least.

(2) i. Simplify $4\frac{1}{9} + \dfrac{1}{\frac{12}{31}} + \frac{13}{18} + \frac{271}{24} + \frac{1}{30}$ of $13 + \dfrac{1}{7\frac{1}{5}}$.

ii. Simplify $1\frac{3}{7}$ of $10 - \frac{3}{14} + \frac{17}{21} - 2\frac{5}{9} + \dfrac{1}{\frac{12}{161}} - \frac{7}{10} - \frac{13}{5}$.

iii. Simplify $\dfrac{1\frac{1}{3} \text{ of } 3\frac{1}{5} \times \frac{15}{28}}{\frac{23}{144} + \frac{5}{18}} + \dfrac{\frac{5}{8} \text{ of } 2\frac{2}{9} \times 1\frac{4}{5}}{\frac{1}{3} \text{ of } \frac{3}{4} \text{ of } 7\frac{2}{5}}$

(3) i. Express in positive terms $\cdot 0148\dot{5}7142\dot{}$ of 2*l.* 14*s.* $8\frac{1}{4}$*d.* + $1\cdot 10\dot{7}1428\dot{5}$ of 3 guineas.

ii. Divide ·008465 by 750, and ·73469 by ·0625.

iii. Find the continued product of $27\cdot\dot{1}4285\dot{7}$, 1·4, and 20·25; also of $16\frac{1}{4}$, $13\cdot\dot{3}$, and 17·125.

(4) i. Express 5 cwt. 2 qrs. 12 lbs. as a decimal of half a ton.

ii. Express in positive terms 5·75 cwt. + ·375 ton + $7\frac{7}{8}$ qrs. + ·0625 lb. + 1·05 ton.

iii. What decimal of $\cdot 838541\dot{6}$ of 2*l.* is $\frac{5}{8}$ of $\frac{3}{4}$ of $1\frac{3}{7}$ of 22 guineas?

(5) i. Bought 5 hhds. of tobacco, each 2 cwt. $17\frac{1}{2}$ lbs. at 3*s.* $4\frac{1}{2}$*d.* per lb., and 7 hhds. of sugar averaging 5 cwt. 2 qrs. 19 lbs. at 2*l.* 13*s.* 8*d.* per cwt.; find the total cost.

ii. What is a gentleman's net income whose salary is 7156*l.* when the income-tax is $10\frac{1}{4}$*d.* in the £?

iii. What must be paid for 27 tons 9 cwt. 2 qrs. 20 lbs. of lead at 18*s.* 8*d.* per cwt.?

(6) i. 75 casks of spirits weighed 725 cwt. 2 qrs. 20 lbs., how many gallons did they contain, allowing 57 lbs. for the weight of each cask, and $1\frac{1}{2}$ lb. to a quart?

ii. Find the net weight of 125 barrels of apples, each 3 cwt. 15 lbs., tare upon the whole 46 cwt. 1 qr. 12 lbs., and tret 4 lbs. in 104 lbs.

iii. Find the net weight of 224 hhds., each 14 cwt. 2 qrs.

14 lbs., tare $9\frac{1}{3}$ lbs. per cwt., tret 4 lbs. in 104 lbs. and draft 2 lbs. in 3 cwt.

(7) i. What sum lent at Simple Interest at $8\frac{1}{3}$ per cent. for 4 years 95 days will 727*l.* 10*s.* amount to?

ii. Compound Interest of 313*l.* 15*s.* for 4 yrs. at $7\frac{1}{2}$ per ct.

iii. What sum lent at Compound Interest at 5 per cent. will amount to 425*l.* 8*s.* $6\frac{21}{40}$*d.* in 5 years?

(8) i. What is the discount of 110*l.* 7*s.* $1\frac{1}{2}$*d.* for $7\frac{1}{2}$ months at $6\frac{1}{4}$ per cent.?

ii. What will a banker gain by discounting a Bill of 2550*l.* drawn March 7th at 10 months, and discounted August 17th at 5 per cent.?

iii. What is the present worth of 1006*l.* 1*s.* $6\frac{3}{4}$*d.* due in $3\frac{1}{2}$ months at $4\frac{4}{5}$ per cent.?

9) i. How much 4 per cent. Stock at 98 can be bought by the transfer of 1960*l.* from the $3\frac{1}{2}$ per cents. at 90?

ii. How much 4 per cent. Stock will realise the same income as 1960*l.* in the $3\frac{1}{2}$ per cents.?

iii. The 3 per cents. realise an interest of $4\frac{1}{2}$ per cent. to the investor, what is the price of Stock?

(10) i. Bought 10 pieces of cloth each containing 25 yds. for 60*l.* 5*s.*; sold 30 yds. at 5*s.* 6*d.* per yd., and 40 yds. at 6*s.*; what must I sell the remainder at per yard so as neither to lose nor gain?

ii. What should I have sold the remainder at to gain 5 per cent. on the whole?

iii. Supposing I had to return 1 per cent. discount to the first two buyers of 30 yds., and 40 yds. respectively; what must I have charged them per yard, so as to have gained 5 per cent. on their purchases?

(11) i. A barters a quantity of linen worth 2*s.* a yard ready money with B, for wine worth 20*s.* a gallon ready money. A puts on 10 per cent. to his ready money price, and there were 210 gallons of wine; how much linen was bartered?

ii. If the wine had turned out inferior, and the linen had been found to be soiled, also if 2*d.* a yard had to be deducted from the barter price, and 3*s*. a gallon from the price of the wine, how would the bargain have to be settled?

iii. A barters sugar for rice, and reckons the price of the latter at 60 per cent. of the former; how many more cwts. of rice will he have than sugar?

(12) i. A, B, C speculate, and gain a sum of money, of which A receives $\frac{1}{6}$, B $\frac{1}{3}$, and C the remainder. If C's capital is 180*l*., what did A invest?

ii. If in (i.) they each invest for a certain number of years, so that at the end of a fixed time they may take equal shares of the gain, when may B and C retire as 'sleeping' partners, and what will be the time of partnership?

iii. Suppose in (ii.) they gain 100*l*. a year, and C retires when he has gained 500*l*., what will be A's share of the gain at that time?

(13) i. A bill of 250*l*. is paid in 5*l*. monthly payments, when should the whole be paid at once?

ii. If in (i.) 50*l*. be paid down, and the rest in 5*l*. payments at intervals of 2 months each, what is the equated time?

iii. If the Governors of Queen Anne's Bounty advance 900*l*. at 4 per cent. to be repaid in yearly instalments of 30*l*., to begin the second year; if the payment in one particular year was 54*l*., how many previous annual payments of interest and capital had been made?

(14) i. Find the square root of ·0000833586620993 1225; also of $\frac{153664}{220900}$.

ii. Find the cube root of ·13 to four figures; and of ·08 to four figures.

iii. A tank whose length, breadth, and depth are alike, is constructed to hold 59730618·429 cubic inches of water; what is the distance round it in yards?

(15) i. Exchange 250*l*. for American dollars, English money being at 7½ per cent. premium.

ii. Exchange 2750 dollars for English money at the same rate as in (i).

iii. A Bill bought in London on Paris at 1*l.* for 25 francs 40 c. is cashed in Amsterdam at $57\frac{1}{2}$ florins for 120 f.; this is sent to Hamburg, where the exchange is 13 marks, 8 schgs. for 11 flor. 50 c.; what is the exchange between London and Hamburg?

PAPER VIII.

Reduce to their simplest form—

(1) i. $\dfrac{15\frac{5}{8}}{1\frac{8}{17}}$ of $\frac{64}{85}\times\dfrac{\frac{11}{27}}{\frac{1}{7}}$ of $8\frac{1}{10}\times\frac{18}{44}$; $\frac{17}{18}\times\frac{19}{20}$ of $\dfrac{6\frac{4}{7}\text{ of }5\frac{1}{7}}{4\frac{3}{5}}\times\frac{13}{51}$ of $\dfrac{6\frac{1}{8}}{\frac{1}{14}}$.

ii. $5\frac{3}{7}+\frac{3}{7}$ of $1\frac{1}{2}+\frac{4}{7}$ of $1\frac{1}{3}+11\frac{17}{28}+2\frac{5}{7}$ of $\frac{4}{5}+\frac{1}{10}$; $7\frac{11}{18}+\frac{14}{15}+\frac{7}{30}+11\frac{17}{48}+\frac{5}{12}+4\frac{3}{4}$.

iii. $7\frac{9}{16}+15\frac{11}{80}-13\frac{1}{2}+25-1\frac{13}{20}+\frac{17}{56}-19\frac{57}{64}$.

(2) i. Add together 3*l.* 7*s.* $11\frac{7}{8}$*d.*, 5*l.* 14*s.* $9\frac{5}{9}$*d.*, 4*l.* 13*s.* $3\frac{1}{2}$*d.* and £$7\frac{4}{21}$, and subtract 9*l.* 19*s.* $11\frac{13}{14}$*d.* from the sum.

ii. A, B, C purchased a ship for 13500*l.*; A paid down ($\frac{7}{15}$ of $1\frac{1}{2}\times\frac{1}{8}$) of the whole, B ($\frac{2}{3}$ of $\frac{1}{2}+\frac{3}{8}$ of $\frac{1}{4}$) of A's, and C $\dfrac{\frac{7}{8}}{2\frac{3}{16}}$ of B's; what did each pay down, and how much remains unpaid?

iii. What fraction of 1857 years are 33 yrs. 121 days 18 hours, reckoning $365\frac{1}{4}$ days to the year?

(3) i. From a cask containing 3 cwt. of coffee I first took out $\frac{1}{3}$ of $\frac{3}{4}+\frac{3}{7}$ of $\frac{2}{9}$ of the whole, and afterwards $\dfrac{\frac{3}{8}}{2\frac{1}{7}}-\frac{3}{20}$ of the remainder; how many lbs. were still left, and reduce them to a decimal of the whole?

ii. Express in positive terms $\cdot 61755\dot{9}523\dot{8}$ of 4 guineas + $5\cdot 325$ of half-a-crown − $\cdot 58125$ of 10*s.*

iii. Express as vulgar fractions $\cdot 023\dot{4}0\dot{8}$; $27\cdot 74\dot{0}09\dot{9}$; $\cdot 21\dot{3}9\dot{6}$.

(4) i. From $\cdot 3745$ take $\cdot 09765$, and multiply the difference by the result of $(7\frac{2}{3}+8\frac{3}{4}+\frac{11}{20})-(3\frac{4}{15}+2\frac{17}{25}+1\frac{19}{30})$.

ii. A mahogany plank is $36\cdot\dot{2}3076\dot{9}$ feet in length and

$3{\cdot}\dot{7}1428\dot{5}$ feet in breadth, what is it worth at 1*s.* 3¾*d.* per square foot?

iii. Find the vulgar fraction equivalent to the sum of $27{\cdot}0\dot{0}\dot{9}$, 6·225, $13{\cdot}1\dot{1}$, $\cdot3\dot{3}$ and 15·75.

(5) i. What will be the expense of repairing two roads, one 95 miles 1 fur. 28 poles in length at 13*l.* 13*s.* 4*d.* per mile, and the other 75 leagues 3 fur. 18 poles at 7*l.* 6*s.* 8*d.* per mile?

ii. Bought 30 casks of tallow, weighing 18 tons 11 cwt. 3 qrs. 26 lbs. at 1*l.* 17*s.* 4*d.* per cwt., paid down ¼ of the total value, and the rest in monthly instalments of 86*l.* 15*s.* 11*d.*; in what time was the whole amount settled?

iii. What is the dividend on 7937*l.* 16*s.* 8*d.* at 18*s.* 6¾*d.* in the pound.

(6) i. Find the net weight of 37 hhds., weighing 310 cwt. 1 qr. 17 lbs., allowing 24 cwt. 1 qr. 17 lbs. for tare, and 1 lb. in 26 lbs. for tret.

ii. If draft (2 lbs. in 3 cwt.) be allowed in the preceding question, find the net value at 3 guineas per cwt.

iii. Find the net weight of 16 hhds. of sugar, weighing 210 cwt. 3 qrs. 21 lbs., allowing 14 lbs. per cwt. for tare, 3 lbs. in 63 lbs. for tret, and 2 lbs. in 300 lbs. for draft.

(7) i. What sum lent at Simple Interest at 4 per cent. will amount to 189*l.* 0*s.* 6¾*d.* in 4 years 3 months?

ii. In what time will the interest of 366*l.* 13*s.* 4*d.* be 93*l.* 0*s.* 10*d.* at 7¼ per cent.?

iii. What sum lent at Compound Interest at 5 per cent. will 910*l.* amount to in 3⅓ years?

(8) i. Find the discount of 184*l.* 15*s.* 1½*d.* for 4 months at 7½ per cent.

ii. What is the present value of 228*l.* 0*s.* 9¼*d.* due in 275 days at 6¼ per cent.?

iii. What did I receive for discounting a bill of 273*l.* 4*s.* 4½*d.* 7 months before it was due at 7 per cent. (True Discount)?

(9) i. A leaves B 3200*l.* in railway shares which are at 5 per

cent. discount; how much will B receive if the legacy duty is 4 per cent. and the charges $\frac{3}{16}$ per cent.?

ii. A invests 5600*l.* in 4 per cent. Stock at $95\frac{1}{4}$, and 333*l.* 6*s.* 8*d.* in $4\frac{1}{2}$ per cent. Stock at 99; he sold the first at 7 per cent. discount, and the second at 2 per cent. premium; what did he realise?

iii. Bought shares at 5 per cent. discount, and sold them at 5 per cent. profit, thereby gaining 14*l.*; how many shares were bought?

(10) i. Bought 1 ton of sugar at 2*l.* 5*s.* 6*d.* per cwt.; at what rate per lb. must I sell it to cover the deduction for tare at 8 lb. per cwt., and gain 10*l.*?

ii. Bought 40 bushels of potatoes at 3*s.* per bushel, but 10 per cent. of them were diseased; what must I sell the remainder for to gain 3*d.* per bushel?

iii. Bought potatoes for 30*l.* 3*s.* at 3*s.* per bushel, some of which were damaged and useless, and sold the remainder at 20 per cent. profit, for which I received 18*l.*; how many bushels were damaged?

(11) i. A has goods which he sold to B for 50*l.* ready money, and he then agrees to take 200 yards of silk at 5*s.* a yard in barter, which was worth 4*s.* 6*d.* ready money. If B will consent to give him as many more yards as will make up the difference between A's ready money and barter price, how many more yards will A have?

ii. How must A and B settle the following transaction?—A gives linen to B for sugar, the prices of which are 2*s.* per yard and 7*d.* per lb. respectively, and the quantities of goods in yards and lbs. are in the inverse ratio of these prices, and there are 210 yds. of linen.

iii. Supposing in the last question A's price per yard had been a ready money price, and B's price per lb. $\frac{1}{7}$ more than his ready money price, what difference would this have made (the quantities of goods bearing the same proportion as before)?

(12) i. If 3 boys do the work of 1 woman, and 5 women that of 3 men, how many boys, women, and men, of each an

equal number, can earn 81*l.* in 6 days, if a boy's hire is 9*d.* a day?

ii. If a number of men, women, and boys earn 486*l.* in 9 days, and the wages are the same as in (i.), how many will there be of each, supposing each set to earn the same money?

iii. If in (ii.) the boys earn $\frac{1}{6}$, the women $\frac{1}{3}$, and the men $\frac{1}{2}$ of the whole sum, how many are there working together?

(13) i. A person is to pay a certain debt by instalments of 5*l.* each every month; the interest of these instalments at 5 per cent. per annum would amount to 17*l.* 18*s.* 9*d.*; what was the debt?

ii. If A owes a debt of 50*l.* which he is to pay by equal instalments of 5*l.* every month, but instead of this he pays it by 10*l.* instalments per month, what does he lose, reckoning interest at 5 per cent.?

iii. Two sums of 50*l.* and 80*l.* are due; the first in 4 months and the second in 6 months; find the sum to be paid in 10 months for which they may be commuted. Interest at 5 per cent. per annum.

(14) i. Extract the square root of $351\frac{469}{2500}$ and of $\frac{289}{1600}$, expressing the results in a fractional and decimal form.

ii. How many yards must I travel in going half round a square field, whose area is exactly 9508015·419049 ft.?

iii. Extract the cube root of 7 to five figures.

(15) i. Exchange 3980 lira at Genoa with Paris, at $99\frac{1}{2}$ lira for 100 francs.

ii. Exchange 200*l.* in Paris, the par of exchange being 25·30 francs per 1*l.*, but French money is at a premium of 10 per mille.

iii. A Bill at Amsterdam is bought in London at 12 florins for 1*l.*, and sold at Hamburg at 13 marks $10\frac{1}{4}$ schgs. per 1*l.*; thence it is sent to Paris and sold at $57\frac{1}{2}$ florins for 120 francs; what was the exchange between London and Paris?

PAPER IX.

(1) i. Simplify $\frac{949}{30680}$; $\frac{18024}{56040}$; $5\frac{3}{50} - 3\frac{17}{30}$; $\frac{11}{18} - \frac{7}{24}$.

ii. Find the value of $\frac{7}{11}$ of $3\frac{7}{9}$ of $5\frac{5}{9}$ of $1\frac{7}{12}$ of $3\frac{3}{4}$ of $\frac{\frac{16}{19}}{3\frac{19}{27}}$ of $\frac{18}{35}$ of 2*l.* 15*s.*, and divide the result by $5\frac{1}{10}$.

iii. Divide the result of $3\frac{1}{9} - 2\frac{1}{18} + 3\frac{3}{8} + 14\frac{9}{16} - \frac{13}{27} - \frac{1}{2}$ by the continued product of 14, $\frac{11}{27}$ of $\frac{1}{12}$, $4\frac{10}{13}$, $3\frac{6}{17}$ of $\frac{19}{124}$ of $7\frac{2}{7}$, $\frac{13}{38}$, and $\frac{14}{57}$.

(2) i. Multiply $\frac{5}{81}$ of $13\frac{7}{8}$ of $1\frac{23}{30}$ of 1*l.* 1*s.* $10\frac{1}{2}$*d.* by $\frac{9}{25}$ of $5\frac{1}{11}$ of $\frac{44}{159}$, and divide the product by $1\frac{10}{27}$ of $\frac{4}{15}$.

ii. If $\frac{5}{6}$ of $\frac{9}{10}$ of $\frac{2}{3}$ of $\frac{7}{8}$ of a house be worth 1662*l.* 10*s.* $8\frac{3}{4}$*d.*, what will be the value of $\frac{4}{9}$ of $\frac{3}{5}$ of $\frac{9}{10}$ of $\frac{5}{8}$ of it? Also, find the value of the house.

iii. From 3 subtract the sum of its fourth, eighth, and twelfth parts; to the remainder add the product of the first and last part; and divide this sum by $18\frac{1}{8}$.

(3) i. A person owes 5*l.* to each of 5 creditors; to the first he pays $\frac{2}{3}$ of his debt, to the second $\frac{1}{4}$, to the third $\frac{1}{6}$, to the fourth $\frac{3}{8}$, and to the fifth $\frac{31}{40}$; what decimal of the whole does he still owe?

ii. Divide $3\cdot07692\dot{3}$ by 1·6; subtract $9\cdot77\dot{2}$ from $12.4\dot{5}$; and find the value of $1\cdot\dot{2}6984\dot{1}$ of 10*s.* 6*d.*

iii. Add together $3\frac{3}{4}$, $\frac{7}{16}$, $11\frac{5}{8}$, $\frac{13}{32}$ and $\frac{123}{40}$, both as vulgar and as decimal fractions; and show that the results agree.

(4) i. Express in positive terms 18·525 of $5\frac{1}{2}$ cwt. + 1·075 of $\frac{1}{2}$ ton + ·00075 cwt.

ii. Find what decimal multiplied by 47 will give the difference between the sum of $7\frac{1}{4}$, $2\frac{1}{3}$, $2\frac{11}{12}$, and the sum of $2\frac{7}{10}$, $3\frac{5}{6}$, $\frac{13}{15}$, $\frac{17}{20}$, $1\frac{5}{8}$.

iii. Simplify $\dfrac{4\cdot\dot{2}8571\dot{4} \times 3\cdot4}{1\frac{3}{10} \text{ of } 2\cdot\dot{4}2857\dot{1}} \times \dfrac{\cdot4\dot{3} \text{ of } \cdot625}{\cdot24}$.

(5) i. Bought 2 hhds. of sugar at $5\frac{1}{2}$*d.* per lb.; one weighed 17 cwt. $23\frac{1}{2}$ lbs., and the other 15 cwt. 2 qrs. $19\frac{3}{4}$ lbs.; what did they amount to?

ii. What must I give for $5835\frac{7}{10}$ cwt. at 3*l.* 5*s.* $1\frac{3}{4}$*d.* per cwt.?

iii. Bought 315 qrs. 7 bus. 1 pk. of malt at 1*l.* 1*s.* 6*d.* per qr., and 710 qrs. 6 bus. 3 pks. at 1*l.* 0*s.* 8*d.* per qr.; find the total cost.

(6) i. What is the net weight of 135 boxes of soap, weighing 713 cwt. 19 lbs., allowing 1 qr. 11 lbs. for the weight of each box? Also the value of the same at $3\frac{3}{4}$*d.* per lb.?

ii. Find the net weight of 73 boxes weighing 118 cwt. 3 qrs. 12 lbs., tare 1 qr. $10\frac{1}{2}$ lbs. per cwt., tret 4 lbs. in 104 lbs. and draft 2 lbs. in 3 cwt.

iii. Find the net weight of 156 boxes, each 7 cwt. 1 qr., allowing 18 cwt. 16 lbs. for the weight of the boxes, and 4 lbs. in 104 lbs. for tret.

(7) i. In what time will the interest on 747*l.* 10*s.* amount to 233*l.* 11*s.* $10\frac{1}{2}$*d.* at $9\frac{1}{8}$ per cent.?

ii. At what rate per cent. will 812*l.* 11*s.* $1\frac{1}{2}$*d.* in 9 yrs. 12 weeks become 1105*l.* 1*s.* $6\frac{9}{25}$*d.*?

iii. What sum will amount to 510*l.* 3*s.* $8\frac{4}{5}$*d.* in 11 yrs. 4 wks. at $5\frac{5}{12}$ per cent.?

(8) i. Find the present value of a debt of 953*l.* 5*s.* $3\frac{3}{4}$*d.* due 10 months hence at $3\frac{1}{2}$ per cent.

ii. Find the difference between the bankers' and true discount of 32*l.* 5*s.* $10\frac{3}{4}$*d.* due in 5 months at $7\frac{1}{8}$ per cent.

iii. Find the bankers' discount on a Bill of 348*l.* 5*s.* 5*d.*, drawn June 17th at 7 months, and discounted October 1st at $6\frac{1}{4}$ per cent.

(9) i. A person invests in $4\frac{1}{2}$ per cent. Stock at 90, and his income from the Stock bought is 36*l.*; find the purchase money.

ii. What is the price of the $3\frac{1}{2}$ per cents. if they yield 4 per cent. on the investment?

iii. Bought Stock at $20\frac{3}{8}$ per cent. discount, sold at 8 per cent. premium, and cleared 340*l.* 10*s.*; how much was invested?

(10) i. Bought 256 cwt. of sugar at 7*d.* per lb., and sold it at 7*l.* 4*s.* 8*d.* per ton; what did I gain?

ii. Bought 456 yds. at 4*s.* 3*d.* per yard, sold $\frac{1}{4}$ of it at 4*s.* 6*d.* per yard, $\frac{1}{8}$ of it at 4*s.* 10*d.*, and $\frac{1}{2}$ of it at 5*s.*; at what price per yard can I afford to sell the remainder to gain 10*l.* by the whole?

iii. Bought cloth at 2*s.* per yd., what must I sell it for to clear 20 per cent., after allowing a discount of 4 per cent.?

(11) i. Three gold chains are given for 5 silver ones; gold is 4*l.* 2*s.* 6*d.* per oz., and silver 5*s.* 6*d.*, and the weight of a silver chain is 12 oz.; what was the weight of a gold one?

ii. A owes B for 12 yds. of silk and 400 yds. of linen. He pays him in tea at 3*s.* 6*d.* per lb., and coffee at 1*s.* 6*d.*, and of each an equal quantity, to the amount of 20*l.*; how much was the linen worth per yard if the silk was worth 3 times as much as the linen?

iii. Suppose in (ii.) the ready money price of the tea and coffee had been 5 per cent. less, what was the ready money price of A's silk per yard?

(12) i. How must A, B, C invest in order that their shares of a gain of 360*l.* may be $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{1}{12}$ respectively, supposing A puts in 108*l.* for 6 months, and B and C put in their money for 2 months and 3 months?

ii. Supposing A, B, C invest as in (i.), and gain 20 per cent. per annum of their capitals, what will each be worth at the end of $2\frac{1}{2}$ years, reckoning at Compound Interest?

iii. In (i.) what is the present worth of C's share of the business?

(13) i. A debt of 100*l.* was due 6 months since, and one of 400*l.* is due 6 months from the present time; when should the two be paid at once, reckoning interest at 5 per cent. per annum?

ii. In (i.) what sum payable in 8 months should the two debts be commuted for?

iii. 100*l.* are due now, 200*l.* were due 4 months since, and 500*l.* are paid by a Bill at 3 months, bearing interest at 4 per cent. per annum; what sum would commute the whole? Interest at 4 per cent.

(14) i. Find the square root of ·03225892589929, and also the

number of acres in a square field whose side is 730·79 yards in length.

ii. Find the cube root of 814806·674960201.

iii. How many cubic feet of stone have been dug from a quarry whose length, breadth, and depth are 16 ft. 1·7 in.? Suppose 27 times as much stone to be obtained from the quarry, and that it is a perfect cube, find its depth.

(15) i. Exchange 810*l.* for dollars when English money is at a discount of 5 per cent.

ii. Exchange 1900 dollars for English money at the same rate as in (i.).

iii. A Bill on Hamburg is bought at 13 marks $10\frac{1}{2}$ schillings per 1*l.* sterling, and sold at Amsterdam at $35\frac{1}{2}$ florins per 40 marks, then laid out in Bills on Genoa at $47\frac{1}{4}$ florins per lira, which are realised at Paris at 1 per cent. discount; what rate of exchange is there between London and Paris?

PAPER X.

(1) i. Find the greatest Common Measure of 3328, 11008 and 296; and the least number that will exactly contain 7, 9, 10, 12, 15, 18, 21, 36, 48.

ii. What number multiplied by the numerator and denominator of the fraction $\frac{125}{927}$ will make it $\frac{3125}{23175}$? also express the fractions $\frac{173}{957}$ and $\frac{53}{163}$ with denominators 69861 and 7009 respectively.

iii. Express as mixed numbers, $\frac{548759}{615}$ and $\frac{93246}{348}$; also express as improper fractions, $434\frac{17}{758}$ and $18307\frac{290}{373}$.

(2) i. Find the product of $3\frac{3}{5}$ of 6 by $3\frac{1}{3}$ of $\frac{11}{15}$, and add to it the quotient of $9\frac{5}{9}$ of $2\frac{1}{6}$ by $\frac{10}{39}$ of $1\frac{1}{6}$. Express the result in a fractional and decimal form.

ii. Find the value of $\dfrac{\frac{19}{27} \text{ of } 5\frac{1}{2}}{\frac{11}{21} \text{ of } 1\frac{11}{27}} \times \dfrac{2\frac{4}{7} \text{ of } \frac{5}{24}}{\frac{5}{19}}$ of 5 guineas.

iii. Find the value of $2\frac{13}{32}l. + 1\frac{5}{12}$ crown + $\frac{11}{18}$ of a moidore ÷ $\frac{5}{7}$

of $2\frac{1}{2}$ guineas + $\frac{7}{12}$ of a shilling, and express the result as a decimal of 10*l.*

(3) i. After paying ·15625 of 100*l.* + ·625 of 5*l.* 5*s.* + 11·575 of 13*s.* $10\frac{1}{2}$*d.*, what had I left out of a 50*l.* note?

ii. Add together 1734 hundredths, 2085 tenths, 27 hundredths, 86 ten-thousandths, 19 hundreds, 3429 hundredths, $1\frac{3}{25}$, 235 and $17\frac{1}{8}$, and subtract the sum from the product of 532·08 by 17·015.

iii. Express as a decimal $\dfrac{\frac{4}{93} \times 2\frac{6}{11}}{1\frac{29}{31} \text{ of } \frac{7}{15}} + \dfrac{\frac{7}{8} \text{ of } 1\frac{11}{21}}{1\frac{1}{3} \text{ of } 9 \times \frac{3}{8}}$.

(4) i. Express $\cdot 135\dot{1}\dot{0}$ and ·312 as vulgar fractions; $10\frac{11}{3125}$ and $\frac{51}{175}$ as decimals; and find the product of ·037846 and ·003698.

ii. Multiply 16*s.* 8*d.* by 2·625; divide 1*s.* 6·75*d.* by ·00015625; and from the sum of these take 13·84475 of 2*l.* 10*s.*

iii. Express in positive terms 7·03125 of a ton + ·0625 of a cwt. + ·15625 of a qr. + 13·125 lbs.

(5) i. A wine merchant imported 427 pipes of wine averaging 108 gallons, at 19*s.* $5\frac{1}{2}$*d.* per gallon, and retailed it at 7*s.* 3*d.* per quart; how much did he gain?

ii. Find the dividend on 4825*l.* at 10*s.* $10\frac{1}{4}$*d.* in the pound, and the loss sustained by a creditor to whom 1327*l.* 10*s.* is due.

iii. Bought 16 barrels of soda weighing 59 cwt. 3 qrs. 17 lbs. at 18*s.* 4*d.* per cwt., and 27 cwt. 2 qrs. 24 lbs. of sugar at 2*l.* 10*s.* 4*d.* per cwt.; find the total cost.

(6) i. Find the net weight of 7 bales of cotton weighing 33 cwt. 1 qr. 20 lbs., tare 14 lbs. per cwt., and tret 4 lbs. in 104 lbs. Also find how much I should lose on the same if cotton fell $\frac{1}{8}$*d.* per lb.

ii. Find net weight of 56 hhds., each 11 cwt. 16 lbs., tare 21 lbs. per cwt., tret 4 lbs. in 104 lbs., and draft 2 lbs. in 3 cwt.

iii. How many half-pound tins can be filled out of 23 boxes of mustard, each 2 cwt. 3 qrs. 15 lbs., allowing 22 lbs. for the weight of a box?

(7) i. Simple Interest of 1205*l.* 10*s.* for 4 yrs. 200 days at $2\frac{1}{2}$ per cent.

ii. Compound Interest of 853*l.* 6*s.* 8*d.* for 4 yrs. at $4\frac{3}{4}$ per cent.

iii. What sum lent at Compound Interest at 5 per cent. will amount to 4084*l.* 2*s.* $0\frac{6}{25}$*d.* in 5 years?

(8) i. Find the discount and present value of a debt of 221*l.* 0*s.* 5*d.* due in 5 months at $7\frac{1}{5}$ per cent.

ii. Find the difference between the bankers' and true discount of 1449*l.* 17*s.* 6*d.* due 9 months hence at $4\frac{3}{4}$ per cent.

iii. Find the discount of 916*l.* 1*s.* $10\frac{1}{2}$*d.* for 1 yr. 10 months at $3\frac{3}{5}$ per cent.

(9) i. If 1500*l.* invested in the $3\frac{1}{2}$ per cents. produce 63*l.* 15*s.* per annum, what is that per cent.?

ii. Transferred 2000*l.* from the 4 per cent. Stock at $94\frac{1}{2}$ to the 5 per cents., and gained £2 $1\frac{150}{2749}$ in income; what was the price of the 5 per cent. Stock?

iii. A person had 3 per cent. Stock at 72, which he transferred to the 4 per cents. at 90, and by so doing increased his income 10*l.*; how much Stock had he in the 3 per cents.?

(10) i. Mixed 10 gallons of wine at 18*s.* a gallon with wine at 25*s.* a gallon; the whole was sold for 34*l.*, thereby gaining 10*l.* 6*s.* 8*d.*; how many gallons of the second wine were there?

ii. Out of 120 dozens of wine I sold enough to cover the cost; the remainder was sold at 10 per cent. profit, and realised 48*l.*, and the quantity sold was to the whole as 4 : 5; what was the cost of the wine?

iii. If I sold wine at 2*l.* per dozen, and gained 5 per cent. on the prime cost, how many bottles out of a pipe containing 25 dozens would cost me nothing?

(11) i. A quantity of cheese at 3*l.* per cwt. and 15*l.* in cash are paid for 24 qrs. of wheat; what was the wheat per bushel, if 4 bushels are equivalent to $1\frac{1}{2}$ cwt. of cheese?

ii. How can A pay B for 12 sacks of flour at 42*s.* per sack, if he has only 1 cwt. of rice at $2\frac{1}{2}$*d.* per lb., and sugar at 4*d.*?

iii. A agrees to take of B linen worth (ready money) 2*s.* 4*d.* a yard, and to pay him with 100 yds. of cloth at 5*s.* 6*d.* a yard (barter price); what quantity of linen did B give to A, supposing B charged 5 per cent. additional to make his charge equal to A's barter price?

(12) i. How long must A, B, C have continued in business to gain 500*l.* each, supposing their capitals are 200*l.*, 300*l.*, and 500*l.* respectively, and not to increase, and they gain 300*l.* per annum altogether?

ii. A and B were in partnership and failed for 3000*l.* A's capital was 750*l.*, and B's 500*l.*, and the liability was limited; what portion would each be liable to, and what dividend would they pay, the assets being 1700*l.*?

iii. A, B, C rent a tract of pasture land at 5*l.* per acre. A puts in 150 sheep, B 50 cows, and C 30 horses. A removes 3 score of his sheep at the end of 3 months, and B adds 20 cows at the end of 4 months. They keep the field for 12 months. If they then pay the same rent, viz. 30*l.* each, what was the proportion between the eating of a cow, a sheep, and a horse? and what was the content of the field?

(13) i. A debt is paid thus: ¼ in 5 months, ½ in 6 months, and the remaining 50*l.* in 8 months; find the equated time.

ii. A debt is paid by a Bill for 100*l.* at 3 months, and by Dock Bonds bearing interest at 4¼ per cent. per annum; and falling due, one in 6 months, and the other in 9 months, the first being ⅔ of the value of the second. The half-yearly dividend of the first was 25*l.* 10*s.*; find the debt, and the equated time for paying it, the rate of interest being 4¼ per cent.

iii. A debt is paid by 3 Bills of 50*l.*, 60*l.*, 70*l.*, due in 3, 4, 5 months respectively, and the interest upon them is 3, 4, 5 per cent. per annum respectively. If the time be extended to 10 months for the payment of the first, at 5 per cent. for the extra time, and the same rate for the others, what would be the amount to be paid at 6 months?

(14) i. What is the length (in yards) of a wall extending round three sides of a square field, whose area is 13 acres 4680 yds. 3 ft. 112·75284681 inches?

ii. How many square yards of sods can be cut from a square field whose side measures 204 yds. 2·91 inches?

iii. Find the cube root of ·000004060773470664.

(15) i. Exchange 6450 dollars into English money which is at a premium of $7\frac{1}{2}$ per cent.

ii. A bill of 150*l.* drawn in London on Paris at 25 f. 40 c, per £1 is cashed at Amsterdam at $57\frac{1}{2}$ florins for 120 francs; how much money is paid in florins at Amsterdam?

A bill of 100*l.* is drawn at Amsterdam at 12 florins per 1*l.*; this is sent to Hamburg at $35\frac{1}{4}$ florins for 40 marks, and cashed again in Paris at 185 f. per 100 marks; how much will it realise in Paris by this route, and what would have been the difference had it been sent direct to Paris at an exchange of 25 f. 30 c. per 1*l.* sterling?

PAPER XI.

(1) i. From $13\frac{3}{20}$ take $12\frac{7}{15}$; add together $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{17}{18}$; multiply $\frac{14}{87}$ of $\frac{1}{5}$ of $\frac{7}{18}$ by $1\frac{1}{28}$ of $3\frac{3}{11}$ of $14\frac{1}{7}$ of $\frac{10}{21}$; and divide $\frac{12}{17}$ of $\frac{5}{18}$ of $6\frac{1}{10}$ of $3\frac{2}{3}$ by $\frac{4}{27}$ of $6\frac{1}{10}$ of $\frac{9}{34}$.

ii. Add together $1\frac{1}{15}$, $2\frac{10}{13}$, $12\frac{1}{65}$, $\frac{1}{8}$; and from the amount take the sum of $5\frac{17}{30} + 2 + \frac{13}{20}$.

iii. Find the value of $\frac{3}{4}$*l.* + $\frac{7}{8}$*s.* + $\frac{11}{12}$ of 13*s.* 3*d.* + $5\frac{3}{5}$ of 2*l.* 10*s.* + $\frac{3}{4}$ of 6*d.*

(2) i. Find the sum of $3\frac{27}{112}$ of a ton, $3\frac{5}{28}$ of a quarter, $\frac{1}{32}$ of a cwt., $\frac{15}{16}$ of a quarter, and $\frac{9}{14}$ of 28 lbs.

ii. A ship worth $10\frac{1}{3}$ of $\frac{17}{18}$ of $\frac{16}{51}$ of $5\frac{2}{3}$ of $\frac{10}{13}$ of $\frac{52}{65}$ of $3\frac{43}{94}$ of 715*l.*, foundered in a storm; find the loss which a merchant will sustain who owned $\frac{3}{4}$ of $\frac{9}{10}$ of $\frac{8}{9}$ of $\frac{7}{8}$ of her; also the value of the vessel.

iii. Simplify $\dfrac{25\frac{3}{4} - 16\frac{1}{2}}{7\frac{1}{4} + 3} \div \dfrac{8\frac{1}{2} \text{ of } \frac{27}{44} \times \frac{11}{51}}{2\frac{7}{8} \text{ of } \frac{32}{69} + 2\frac{8}{23}}$.

(3) i. From a purse containing 20*l.* 16*s.* 8*d.* I took $\left(\dfrac{5\frac{1}{5} + 2\frac{1}{3}}{2\frac{3}{5} \text{ of } \frac{3}{7}} + \dfrac{7\frac{7}{10}}{1\frac{1}{3} \text{ of } 11}\right)$ of the third of it; how much remained, and what decimal of the original amount is it?

ii. Divide 4·725 of a penny by ·007875; multiply 3*l.* by 4·3125, and reduce the former to a decimal of the latter to 8 figures.

iii. Express in positive terms ·625 of a ton + 156·25 of a cwt. + ·375 of a quarter + 3·5 of a lb. + ·25 of an oz.

(4) i. What vulgar fraction is equivalent to the sum of $17\cdot3\dot{3}$, 3·06, $\cdot2\dot{3}$, 9·25, and $13\cdot1\dot{8}$?

ii. What decimal of 3 guineas is 2*l.* 19*s.* 7½*d.*, and how often is 29·7 contained in 253·35?

iii. Reduce to a simple quantity $\left\}\dfrac{6\cdot6041\dot{6}}{33\cdot0208\dot{3}} + \dfrac{13\cdot\dot{3} \times \cdot375}{3\frac{5}{7} \text{ of } \cdot\dot{5}3846\dot{1} \text{ of } \frac{3}{4}} + \dfrac{2\cdot7\dot{7}\dot{2}}{11\cdot\dot{0}\dot{9}}\right.$.

(5) i. The rental of a nobleman's estate is 6825*l.* per annum; find the yearly outgoings arising from a poor rate of 2*s.* 4¼*d.*, a land tax of 1*s.* 3½*d.*, and a church rate of 1½*d.* in the pound respectively.

ii. Find the value of 17½ chests of tea, each weighing 3 qrs. 14 lbs., at 3*s.* 11¼*d.* per lb.

iii. Bought 92 French ells 2 qrs. 3 nls. of cloth at 11*s.* 2*d.* per ell, and 14 English ells 4 qrs. 3 nls. at 14*s.* per yd., what did they both amount to?

(6) i. Find the net value of 43 boxes weighing 523 cwt. 1 qr. 14 lbs., tare 2 qrs. 16 lbs. per box, at 2*l.* 15*s.* per cwt.

ii. What is the net weight of 127 barrels, each 5 cwt. 2 qrs. 12 lbs., tare 18 lbs. per cwt., and tret 4 lbs. in 104 lbs.?

iii. Bought 39 bags of sugar, each 3 cwt. 2 qrs. 22 lbs., tare 13⅓ lbs. per bag, and tret 4 lbs. in 104 lbs.; how many parcels of 25 lbs. each could be made out of it?

(7) i. In what time will 110*l.* 5*s.* 10*d.* amount to 152*l.* 0*s.* $10\frac{11}{14}$*d.* at 5 per cent.?

ii. What principal will amount to 170*l.* 16*s.* $5\frac{1}{4}$*d.* in 4 years 5 months at $4\frac{1}{2}$ per cent.?

iii. What sum will 2500*l.* at Compound Interest at 6 per cent. for $3\frac{1}{2}$ years amount to?

8) i. Find the discount of 800*l.* 14*s.* 6*d.* due in 8 months at $6\frac{1}{4}$ per cent.

ii. I hold a bill of 1900*l.* 10*s.* 6*d.* which is due in 7 months, but I wish to have it cashed; what will the banker give me for it, and how much less is it than what I should really have?

iii. On the 24th of July I had a bill of 965*l.* 14*s.* 7*d.* discounted at $5\frac{1}{2}$ per cent. It was drawn on the 3rd of June at 6 months; what did I receive from the banker?

(9) i. A has 6400*l.* in London and North Western Stock, which he bought at 104. He received 6 half-yearly dividends, at $4\frac{1}{4}$ per cent. per annum for the first two, and an average of $5\frac{3}{4}$ per cent. for the remaining dividends; how much did he receive altogether?

ii. Bought railway shares at $85\frac{1}{2}$, paid commission and expenses amounting to £$\frac{1}{8}$ per share; received the half-yearly dividend (at the rate of 4 per cent. per annum) two months after the purchase; what was the rate per cent. per annum on my capital?

iii. If in the last question I sell out at 86 two months before the next dividend, how much will I have gained by the transaction, reckoning interest at 4 per cent. per annum?

(10) i. Bought 256 cwt. of sugar, at 3*l.* 3*s.* 6*d.* per cwt., and sold it at $7\frac{1}{2}$*d.* per lb.; how much did I gain, supposing I had to make allowance for tare of 7 lbs. per cwt.?

ii. Currants at 4*d.* per lb. are mixed with 24 lbs. at 6*d.*, 48 lbs. at 9*d.*, and 96 lbs. at 11*d.*; the mixture is sold at $10\frac{1}{2}$*d.* per lb., and the gain is 50*s.* on the whole; how many lbs were there at 4*d.*?

iii. Wine at 20*s.* per gallon was mixed with others at 30*s.*

and 40*s.* per gallon each. If the mixture was in the proportion of 3 : 2 : 1, and if it was sold at 5 per cent. profit, find what the gain or loss per cent. would be if the proportion had been 1 : 2 : 3, and the wine had been sold at the same price.

(11) i. A buys 20 cwt. of cheese from B, and agrees to pay in 6 months, and 4 per cent. interest on the bill for that time. Instead of this, A barters his crop of hay to be delivered in 3 months. There were 13 tons of hay at 6*l.* 10*s.* a ton; what was the cheese per cwt.?

ii. A exchanges tea for meat at 9*d.* per lb. with B, and when 50 lbs. of meat had been supplied, A had given $\frac{3}{8}$ of the tea; how much meat did B give?

iii. In (ii.) if 1 lb. of meat had been worth $\frac{3}{20}$ of 1 lb. of tea (ready money), and the worth of 2 oz. of tea had been put on to the lb. as the barter price, what would have been the barter price of the meat?

(12) i. A, B, C are allotted shares in a railway, upon which a call of 10 per cent. is paid on allotment; if they are 20*l.* shares, and A pays $\frac{1}{4}$ as much again as B, and B $\frac{1}{2}$ as much as C, what number of shares were allotted to each, if A pays 150*l.*?

ii. If in (i.) a further call of 15 per cent. of the remainder of the liability is made in 3 months, and A has failed, what sums must B and C pay to make up their own shares, together with A's, each in proportion to his number of shares?

iii. Suppose in (ii.) the company pays a dividend of 10 per cent. on the paid-up capital, how much will B and C be out of pocket by A's failure, if they take his shares? Supposing they received the same dividend for 10 years, what would be the result of their liability in a pecuniary point of view, no fresh calls being made?

(13) i. If I settle a Bill of 100*l.* due in 10 months by 3 bills of credit, one for 3 months, and two others of 30*l.* and 50*l.* respectively at 4 months, what discount shall I get at 4 per cent.?

ii. Three debts of 41*l.*, 60*l.*, 100*l.* are due in 4, 5, 6 months respectively. If I give a Bill for 40*l.* due 6 months hence, and bearing interest at 5 per cent. at once, what sum must I pay in 8 months to commute the remaining debt, interest at 4 per cent. per annum?

iii. A debt of 260*l.* is paid by instalments of 5*l.* each per month, and the interest on the debt remaining (at 4 per cent. per annum) after each payment; how much will be paid altogether?

(14) i. Find the square root of 23219 to 5 decimal places, and the area of a square, the distance round which is 68·38 yards.

ii. Find the cube root of 405322575·992216.

iii. What is the cost of lining with lead, at 1*s.* 3*d.* per square foot, a tank whose length, breadth, and depth are equal, and which will contain 136519·19 cubic feet of water?

(15) i. If 600 dollars realise in New York 125*l.*, and the par value of a dollar is 4*s.* 6*d.*, how much per cent. is American money depreciated?

ii. If 104*l.* buy 480 dollars in New York, what is the premium on English money?

iii. A Bill of 600*l.* drawn on New York is sold in Paris at a discount of 5 per cent. on the exchange price; it is converted into Paris bills in francs, which are sent in payment to New York at an exchange of 5 francs for 1 dollar. If English money be 2 per cent. premium in Paris, what amount would they realise in New York, where English money is at 5 per cent. premium, and 25 francs being equivalent to 1*l.* at par?

PAPER XII.

(1) i. Find the highest common divisor of 6279, 20769, 15801 and 12581; and the L. C. M. of 6, 8, 7, 12, 15, 17, 20, 24, 28, 34.

ii. Express $\frac{343}{7003}$ as a fraction, with 112048 for its denom nator; $\frac{746382}{544}$ as a mixed number; $3437\frac{13}{945}$ as an improper fraction; and simplify $\dfrac{\frac{56}{57}}{1\frac{2}{19}}$.

iii. Find the sum, difference, product, and quotient of $\frac{21}{44}$ and $\frac{16}{33}$, expressing each result in a fractional as well as a decimal form.

(2) i. Add together $2\frac{3}{4}$, $\frac{5}{6}$, $1\frac{11}{12}$, $\frac{7}{10}$, $15\frac{11}{20}$, and $\frac{8}{15}$; multiply the sum by $\frac{4}{21}$, and see how often the product is contained in $16\frac{44}{45}$.

ii. Distributed 672*l.* among four persons; to the first $\frac{2}{3}$ of $\frac{7}{8}$ of the whole, to the second $\frac{4}{5}$ of $\frac{7}{8}$ of the remainder, to the third $\frac{1}{3}$ of $\frac{6}{7}$ of what was then left, and the rest to the fourth. Find what fraction each had of the whole, and its value.

iii. Simplify $\left\{\left(\frac{14}{15}-\frac{11}{45}\right)+\left(2\frac{1}{12}-1\frac{13}{27}\right)\right\}+\left(\dfrac{\cdot625\times\cdot8\dot{6}\text{ of }\frac{21}{28}}{1\frac{1}{16}\text{ of }1\cdot\dot{1}4285\dot{7}}\div\dfrac{1\frac{1}{34}}{\cdot9\dot{5}\times\frac{20}{57}}\right)$

(3) i. What fractional part of *5l. 5s.* is *2l.* 13*s.* $10\frac{1}{2}$*d.*; and what decimal part of 500*l.* is 1*s.* $6\frac{3}{4}$*d.*?

ii. Find the value of 13·15625*l.* + ·000625 of 20*l.* + ·8325 of 13*s.* 4*d* − ·6845 of 9*l.* 12*s.* 3*d.*

iii. Simplify $\left\{\dfrac{\cdot3\dot{7}1428\dot{5}}{5\cdot2}+\left(\dfrac{4\cdot75}{6\cdot\dot{3}}\div\dfrac{5\frac{1}{2}-\cdot75}{2\cdot\dot{7}1428\dot{5}}\right)\right\}\div\left(\dfrac{9\cdot\dot{1}\dot{8}}{5\frac{16}{17}}+\dfrac{\frac{2}{3}\text{ of }\frac{3}{4}\text{ of }\cdot8\dot{3}.}{\frac{1}{12}}\right)$

(4) i. Add together $13\cdot\dot{3}$, 26 ninths, $11\cdot\dot{2}8571\dot{4}$ and 11·25.

ii. Divide $\cdot09\ddot{8}6\dot{4}06$ by 625; and find the value of $\dfrac{8l.\ 13s.\ 8\frac{1}{2}d.}{2\cdot36875}$.

iii. The sum of 368*l.* 17*s.* $6\frac{3}{4}$*d.* is to be divided among five persons, so that the first has 25 shares, the second 23, the third 21, the fourth 17, and the fifth 13; find the value of each share, and what decimal it is of the whole.

(5) i. Find the difference between the rent of two farms, one comprising 127 acres 2 roods 29 poles at 2*l.* 3*s.* 4*d.* per

acre, and the other 159 acres 32 poles at 1*l.* 16*s.* 8*d.* per acre.

ii. Find the value of 8 tons 15 cwt. 3 qrs. 17 lbs. at 1*l.* 17*s.* 4*d.* per cwt.

iii. Bought 45 hhds. of sugar, weighing net 187 cwt. 3 qrs. $17\frac{1}{4}$ lbs. at $5\frac{1}{2}$*d.* per lb.; find the total cost.

(6) i. Find the net weight of 319 bales, each 17 cwt. 2 qrs., tare 1 qr. 14 lbs. per bale, and tret 4 lbs. in 104 lbs.

ii. Bought 137 barrels of petroleum oil, each $10\frac{1}{2}$ cwt., whole tare 86 cwt. 12 lbs.; find the net weight, and also the number of gallons of oil, allowing 8 lbs. to the gallon.

iii. Imported 70 bags of sugar, weighing 133 cwt. 2 qrs. 24 lbs., tare $7\frac{7}{15}$ lbs. per cwt., tret 4 lbs. in 104 lbs., and draft 2 lbs. in 3 cwt.; find the net value at $3\frac{1}{4}$*d.* per lb.

(7) i. At what rate per cent. will the interest of 454*l.* 13*s.* 9*d.* become 244*l.* 0*s* $3\frac{3}{4}$*d.* in 7 years 8 months?

ii. What sum lent at Simple Interest for 6 yrs. 18 wks. at 5 per cent. will amount to 862*l.* 0*s.* $3\frac{9}{32}$*d.*?

iii. What sum lent at Compound Interest for 3 yrs. at $7\frac{1}{2}$ per cent. will amount to 1135*l.* 2*s.* $11\frac{451}{640}$*d.*?

(8) i. What is the present worth of a Bill of 338*l.* 16*s.* $10\frac{1}{2}$*d.* due in 7 months at $5\frac{3}{5}$ per cent.?

ii. Find the difference between the bankers' and true discount of 300*l.* 2*s.* 6*d.* for 4 months at $6\frac{1}{4}$ per cent.

iii. On July 1st 1864, I received a bill of 846*l.* 15*s.* due in 12 months, but on Nov. 1st of the same year I took it to the Bank, to be discounted; how much did I receive, the rate of discount being $4\frac{3}{4}$ per cent.?

9) i. Which is the best Stock to buy, the 3 per cents. at 80, or the $4\frac{1}{2}$ per cents. at 85?

ii. By how much would my income be altered by transferring 500*l.* from the $4\frac{1}{2}$ per cents. at 81 to the 3 per cents. at 75?

iii. Bought 7000*l.* in the 3 per cents. at 75, sold out 4000*l.* at 77, and the remainder at a loss of 2 per cent. on the cost; what is gained or lost by the transaction?

(10) i. I buy 247 yds. of cloth at 2*s.* 4*d.* a yard; how must I sell it per yard to gain 4*l.* 2*s.* 4*d.* on the whole?

ii. Bought 876 yds. for 328*l.* 10*s.*; what must I sell it at per English ell to gain 65*l.* 14*s.* on the whole?

iii. A tradesman gains $3\frac{1}{2}$*d.* on an article which cost him 1*s.* 2*d.*; what is the gain per cent.?

(11) i. A has a cow, two pigs, and a horse, and values them in barter to B for 3, 1, 7 sheep respectively; find the value of A's exchange, if a pig is worth 30*s.*

ii. If a sheepskin is worth $\frac{1}{30}$ of the sheep, and the wool realises 5 per cent. less than the skin, then in (i.) by how much is A's bargain better than B's, supposing these circumstances are considered?

iii. A agrees to exchange linen for cloth with B; the linen was to be 2*s.* per yard, but being soiled is bartered at its cost price. The cloth was inferior to the sample, and was taken at its cost, i. e. $\frac{3}{4}$ its barter price, which was 8*s.* a yard; and the ready money price of A's linen was 20*l.* How many yards of linen, and also of cloth, were there, supposing the former to undergo a proportionate increase in barter to the latter?

(12) i. A, B, C speculate and gain each 100*l.*, but A's money had been in trade 12 months, B's 6 months, and C's 3 months; what were their capitals if the gain was 20 per cent. per annum of their investments?

ii. A and B trade together, and gain at the end of 12 months from B's commencing business, 30*l.* and 60*l.* respectively. A's money was twice the amount of B's; when did A join B?

iii. A has $\frac{2}{3}$ of the capital of a concern, and B and C have each $\frac{1}{8}$ of the profits for their services. If the profits are 9000*l.* per annum, and are found to be a gain on the capital of 5 per cent., what will each of them receive if they first pay 6 per cent. per annum for the $\frac{1}{3}$ of the capital they had borrowed?

(13) i. A owes B 500*l.*, of which $\frac{1}{2}$ is payable in 6 months, $\frac{1}{3}$ in

9 months, and the rest in 18 months; find the equated time.

ii. A owes B 200*l.* to be paid in 8 months; if he pays 40*l.* down, and 100*l.* in 7 months, when should he pay the remainder?

iii. If $\frac{1}{2}$ of a debt of 600*l.* is paid in 6 months, and $\frac{1}{3}$ in 9 months, when should the remainder be paid if the equated time for paying the whole be 9 months?

(14) i. Find the square of 734691, and the square root of 87798284·89146121.

ii. Find the cube of 7507, and the cube root of 348467799653.

iii. Find the area of the side of a cube whose solidity is 13634·789869 feet.

(15) i. Exchange 562*l.* 10*s.* for American dollars at 4*s.* 6*d.* per dollar.

ii. Also if English money is at a premium of $7\frac{1}{2}$ per cent.

iii. Also if English money is at a discount of 5 per cent.

PAPER XIII.

(1) i. Find the greatest number that is contained in each of the numbers 972, 2673, 7857, and 14769.

ii. Find the least sum of money which contains an exact number of either moidores, guineas, sovereigns, crowns, florins, or marks.

iii. Add together $14\frac{1}{7}$, $\frac{16}{21}$, and $3\frac{1}{3}$; subtract from this result the sum of $11\frac{2}{7}$ and $\frac{4}{9}$, and divide the difference by the sum of $13\frac{3}{4}$, $\frac{7}{10}$, $11\frac{5}{8}$, and $19\frac{173}{360}$.

(2) i. Divide the continued product of $13\frac{3}{5}$, $\frac{17}{18}$, $3\frac{7}{10}$, $\frac{11}{17}$, $\frac{15}{16}$, and $1\frac{1}{17}$ of 10 of $\frac{5}{6}$ by $25\frac{7}{16}$.

ii. Add together $\frac{17}{32}$ of a ton, $2\frac{3}{4}$ of half a cwt., $7\frac{1}{8}$ of a qr., $\frac{5}{7}$ of a cwt. and $\frac{3}{4}$ lb.

iii. A gentleman left $\frac{1}{2}$ of his property to his wife, $\frac{1}{3}$ of the remainder to his eldest son, $\frac{1}{5}$ of what was then left to each of his other two sons, to each of his three daughters $\frac{1}{5}$ of the eldest son's share, $\frac{2}{3}$ of what still remained equally among four grandchildren, and the remaining 800*l.* to various charities. Find the value of his property, also of each person's share.

(3) i. Add together $4\cdot\dot{2}$, $7\cdot208\dot{3}$, $13\frac{1}{48}$, and from the sum subtract $1\cdot08\dot{3} + 2 + \frac{29}{84}$, expressing the result in a fractional as well as a decimal form.

ii. How much was left out of a 10*l.* note after paying ·375*l.*, 1·25 of 13*s.* 4*d.*, 12·625 of 10*s.*, and ·3125 of a crown ?

iii. What did a watch cost whose value if multiplied by 5·003125 would give 52*l.* 10*s.* 7·875*d.* ?

(4) i. Find the sum of 39·3729, 173 tenths, 3469, 5 tenths, 346 thousandths, 89 hundred-thousandths, 235·748, 7345, 187·594, 25 tenths, 34 hundredths, 68 ten-thousandths, 1392 tenths, and from it subtract the product of ·003468 by 379 ten-thousandths.

ii. Express in positive terms 19·625 of a ton + 3·125 of a cwt. + 2·025 of $3\frac{1}{2}$ qrs.

iii. Reduce $\left(\frac{17\cdot\dot{1} \times 1\cdot\dot{6}\dot{3}}{4} \times \cdot10\dot{7}1428\dot{5}\right) + \left(\frac{1\cdot4 \times 20\cdot25}{\frac{7}{135}}\right) - \left(\frac{\cdot\dot{4}2857\dot{1}}{\cdot0\dot{1}\dot{7}1428\dot{5}} + \frac{724\cdot37\dot{5}}{2.318}\right)$ to its simplest form.

(5) i. A pack of flour weighs 240 lbs. ; find the cost of 59 packs at 2*s.* $5\frac{1}{4}$*d.* per stone of 16 lbs.

ii. What must be paid for the carriage of 325 tons of iron ore a distance of 257 miles 6 furlongs 27 poles at 4*l.* 13*s.* 4*d.* per mile ?

iii. What sum must be handed over to the owner of an estate which sells for 7687 guineas, the auctioneer's and other expenses being 1*s.* $11\frac{1}{4}$*d.* in the pound ?

(6) i. Find the net weight of 26 hogsheads weighing 125 cwt. 14 lbs., tare 16 lbs. per cwt., tret 4 lbs. in 104 lbs.

ii. How many canisters, each containing 30 lbs., can be filled from 57 bags of coffee weighing 346 cwt. 2 qrs. 17 lbs., allowing 16 lbs. for the weight of each bag ?

iii. In one week 2000 barrels of oil weighing 124 tons 16 cwt. were imported. Find the net value of this at $7\frac{3}{4}$*d.* per quart, allowing 1 qr. $3\frac{1}{2}$ lbs. per cwt. as tare, 4 lbs. in 104 lbs. for tret, and 2 lbs. in 3 cwt. for draft, and 2 lbs. to a quart.

(7) i. At what rate per cent. will 1112*l.* 10*s.* amount to 1512*l.* 6*s.* $1\frac{1}{8}$*d.* in 5 years 39 weeks?

ii. In what time will the interest of 327*l.* 17*s.* 6*d.* become 71*l.* 6*s.* $3\frac{3}{40}$*d.* at $4\frac{1}{2}$ per cent.?

iii. Compound Interest of 864*l.* 7*s.* 6*d.* for 4 years at $6\frac{1}{4}$ per cent.

(8) i. What is the true discount of 1449*l.* 17*s.* 6*d.* due in 9 months at $4\frac{3}{4}$ per cent.?

ii. What is the true present worth of a bill of 1060*l.* 10*s.* $0\frac{1}{4}$*d.*, which has 73 days to run, at $4\frac{1}{2}$ per cent.?

iii. What does a banker receive more than his due on a bill of 3237*l.* 10*s.*, drawn January 3, at 10 months, and discounted April 1, at 6 per cent.?

(9) i. A invests 550*l.* 3*s.* 4*d.* in the $3\frac{1}{2}$ per cents. at 75, and B 250*l.* 6*s.* 8*d.* in the 4 per cents. at 80; what sum must B invest in the 5 per cents. at 105 that his income may be equal to A's?

ii. A person invests 450*l.* in French 5 per cent. stock at 95, and sells out at a loss of 2 per cent. of French money on his purchase; what does he lose supposing 1*l.* = 25·25 francs when he bought and 25·75 francs when he sold out?

iii. A person gets a half-yearly dividend of 31*l.* 13*s.* 9*d.* (after paying an income-tax of 6*d.* in the pound) from a 5 per cent. stock; how much has he invested? and supposing it paid him an interest of $6\frac{1}{2}$ per cent. on his investment, at what price did he buy it?

(10) i. Bought tea at 3*s.* 6*d.* per lb. and sold it at 4*s.* 8*d.*, thereby gaining 7*l.*; how many lbs. did I buy?

ii. 10 hogsheads of sugar are bought at 2*l.* 2*s.* per cwt. and sold at 5*d.* per lb.; if they realised 300*l.*, how much did they weigh, and what was the profit?

iii. What is gained or lost per cent. by purchasing at 2*l.* 10*s.* per cwt. and selling at 9*d.* per lb.?

(11) i. A wine merchant has 3 butts of sherry, each 120 gallons,

the prices being 14*s.*, 18*s.* and 20*s.* per gallon respectively. He takes in exchange pale ale in barrels of 36 gallons at 1*s.* 4*d.* per gallon; how many barrels of ale does he get?

ii. Supposing in (i.) he had mixed a butt of water with his wine in equal proportions, how much per gallon would he charge to obtain the same value for his wine if he reckoned water at 1*s.* per gallon, and how many barrels of ale at a price 5 per cent. higher?

iii. A has 200 yards of linen worth 2*s.* 6*d.* a yard, which he barters with B at 2*s.* 7½*d.* a yard, taking in return linen at 3*s.* 9*d.* a yard, which B sold at 3*s.* 4*d.* ready money; how much linen did A receive?

(12) i. A, B, C own a ship. A paid for ½, B for $\frac{5}{12}$, C for $\frac{1}{12}$. The ship and cargo, valued at 72,000*l.*, are totally lost; find the loss of each.

ii. Suppose in (i.) the cargo had been insured for 20,000*l.*, which was 80 per cent. of its value, and the wreck had been saved and sold for 4 per cent. of its cost; what would then have been the loss of each?

iii. A, B, C gain 3200*l.* in two years; A advances ½ of the capital, B ¼, C ⅛, and they borrow the remainder (240*l.*) in equal proportions; what will each receive supposing they pay 5 per cent. for the borrowed portion?

(13) i. Two sums of 200*l.* and 350*l.* are due in 4 months and 5 months hence respectively; find the equated time.

ii. In (i.) find the sum to be paid in 10 months, which will commute them, interest being at 5 per cent.

iii. In (i.) suppose the first debt had been over due for 12 months, find the sum to be paid to commute them in 10 months.

(14) i. Find the square root of ·00490140582681967281, and the number of yards round a square whose area is 496898·1081 feet.

ii. The total cost of laying down sods on a square cricket field was 2597*l.* 2*s.* 6¼*d.*, at 1*d.* per square yard; what was the length of one of its sides?

iii. Find the cube root of 131197182717797.

(15) i. Exchange 850*l.* for francs at 25·15 francs per 1*l.*

ii. Exchange 6785·11 francs for English money at 23·50½ francs per 1*l.*

iii. If English money is at a premium of 5 per cent. in Paris, how many francs would there be for 850*l.*, at 25·15 francs, exchange at par?

PAPER XIV.

(1) i. Express $59\frac{213}{527}$ and $13469\frac{13}{995}$ as improper fractions; also $\frac{742868}{576}$ and $\frac{3684235}{725}$ as mixed numbers.

ii. Reduce to their lowest terms $\frac{9744}{107424}$, $\frac{54480}{728448}$, $\frac{8023}{421101}$.

iii. Find the value of $\frac{17\frac{2}{3}}{2\frac{3}{5}}$ o $\frac{5}{9}$ of $\frac{\frac{11}{12}\text{ of }2\frac{2}{53}}{3\frac{1}{2}\text{ of }\frac{19}{26}}$ of $1\frac{1}{8}$ of $3\frac{5}{11}$ of 100 guineas.

(2) i. Add together $5\frac{2}{7}$, $13\frac{3}{14}$, $\frac{13}{21}$, $51\frac{2}{35}$, $\frac{31}{42}$; and divide the sum by the continued product of $1\frac{8}{9}$, 6, $2\frac{14}{15}$, $2\frac{4}{7}$, $\frac{3}{22}$, and $\frac{5}{68}$.

ii. From the sum of $\frac{11}{18}$, $11\frac{1}{7}$, $17\frac{17}{36}$, and $\frac{1}{2}$, take the sum of $\frac{2}{15}$, $\frac{19}{21}$, and $12\frac{2}{5}$, expressing the result in both a fractional and decimal form.

iii. Express the sum of $\frac{7}{32}$*l.*, $10\frac{5}{8}$ of a shilling, $\frac{14}{15}$ of a crown, $2\frac{9}{14}$ of a guinea, and $\frac{7}{10}$ of 16*s.* 8*d.* as a fraction of 5*l.*

(3) i. Simplify $\frac{4\frac{1}{4}\text{ of }\frac{3}{8}\text{ of }\frac{16}{17}}{\frac{1}{20}\text{ of }5} - \frac{\frac{3}{8}\text{ of }\frac{7}{10}}{2\frac{2}{7}\text{ of }\frac{3}{5}\text{ of }\frac{1}{6}}$, expressing the result as a decimal.

ii. Express as a decimal $\left(\frac{\frac{3}{20}\times\cdot\dot{7}\times\cdot 7}{1\cdot 19\dot{4}\times\frac{7}{30}}+\frac{1\frac{1}{25}\text{ of }\frac{5}{43}}{3\cdot\dot{4}2857\dot{1}}\right)+\left(\frac{2}{3\cdot\dot{3}}\div\frac{\cdot\dot{2}8571\dot{4}}{8\frac{2}{5}}\right)$.

iii. From the sum of 17·025*l.*, $7\frac{3}{8}$*l.*, and 72·325*l.* take the sum of 13·0$\dot{6}$*l.*, $29\frac{5}{9}$*l.*, and 11·$\dot{6}$*l.*; how often is the difference contained in 250*l.* 19*s.*?

(4) i. Express 17*s.* $10\frac{3}{4}$*d.* as a decimal of 5*l.*; and find the value of $\cdot 61755\dot{9}523\dot{8}$ of 4*l.* 4*s.*

ii. Simplify $\frac{783.6}{89{\cdot}55\dot{4}2857\dot{1}} + \frac{3{\cdot}6 \times 7{\cdot}1\dot{6} \times \frac{25}{26} \text{ of } 4{\cdot}75}{\frac{1}{10} \text{ of } 7{\cdot}\dot{6}9230\dot{7} \text{ of } 7\frac{3}{7} \times 3}$ giving the result in both a fractional and decimal form.

iii. A, B, C, D buy a ship: A pays ·375 of the whole; B ·6 of what A paid; C ·625 of the remainder; and D 3708*l.* Find the value of the ship, also of each person's share.

(5) i. Find the value of $7341\frac{7}{12}$ cwt. of soda at 1*l.* 16*s.* 5½*d.* per cwt.

ii. Find the value of 18 cwt. 2 qrs. 23 lbs. of lead at 30*l.* 6*s.* 8*d.* per ton.

iii. Find the dividend on 5015*l.* at 13*s.* 5¾*d.* in the pound, and the loss sustained by a creditor to whom 626*l.* 17*s.* 6*d.* is due.

(6) i. Find the net weight of 200 bales of cotton, each 5 cwt. 18 lbs., allowing 25 cwt. 1 qr. 12 lbs. for tare.

ii. How many gallons of oil were contained in 133 casks, each 3 cwt. 1 qr. 26 lbs., and the weight of the cask 1 qr. 13 lbs., allowing 7 lbs. to the gallon?

iii. Find the net value of 57 barrels weighing 237 cwt. 2 qrs. 24 lbs., tare 19¼ lbs. per cwt., and tret 4 lbs. in 104 lbs., at 4*l.* 11*s.* per cwt.

(7) i. What principal will amount to 1868*l.* 7*s.* 0½*d.* in 5 years 3 months at 3½ per cent.?

ii. Find the Compound Interest of 813*l.* 13*s.* 4*d.* for 4 years at 5 per cent.

iii. What sum will 1112*l.* 10*s.* amount to at Compound Interest, at 5 per cent. for 3½ years?

(8) i. Find the discount and present value of 760*l.* 0*s.* 7½*d.* due in 10 months at 3¾ per cent.

ii. Find the difference between the bankers' and true discount of 365*l.* 10*s.* 3*d.* for 15 months at 5½ per cent.

iii. On the 3rd of April I received a Bill of 1100*l.* 15*s.*, which had 5 months to run; what was it then worth, reckoning the rate of discount at 6 per cent.?

(9) i. If money in the 4 per cents. pays an interest of 2*l.* 3*s.* 9*d.*, what is the price of Stock?

ii. If the price of Stock is 96 and the interest on the money invested realises 4*l.* 10*s.* per cent., find the rate of interest of the Stock.

iii. Bought railway shares of 10*l.* each at 10 per cent. discount, and sold them at par, thereby gaining 15*l.*; how many shares did I buy?

(10) i. Cloth is bought at 5*s.* a yard and sold at 6*s.* per English ell; how much will be gained or lost in selling 750 English ells?

ii. If I am allowed 10 per cent. discount upon the price of an article, and I sell it at 5*s.* 6*d.*, thereby gaining 10 per cent., what was the original price of it?

iii. If I sold an article for 10*s.* and gained 6 per cent. on the cost, what should I have sold it for to have gained 15 per cent.?

(11) i. A charges B for cheese, in barter, 2 per cent. more than B charges for linen, ready money. If A gives B 20 cwt. of cheese for 200 yds. of linen, which B sells at 4*s.* 6*d.* per yard ready money, what did A charge per cwt. for his cheese?

ii. Suppose in (i.) the cloth had been half as cheap, and the cheese half as dear again in barter, what would have been the equivalents in cheese and cloth, if business to the amount of 30*l.* had been transacted between them?

iii. A charged 20*l.* for the cheese sold to B, and B gave 10*l.* in cash and the rest in linen at 2*s.* 6*d.* per yard; but as A's goods were inferior to the sample, B deducted 10 yards of his linen; what was the cheese actually sold at per cwt. if 14 yds. of linen were equal in value to 1 cwt.?

(12) i. A puts into a concern 400*l.*, and B, 3 months after, 200*l.*; 6 months after this they both put in 200*l.* each, and they gain 250*l.* in 1½ year; how must they share it?

ii. A, B, C rent a field for 10 months at 1*l.* 10*s.* per month, and A puts in 10 cows for 8 months; how many can B put in for the remainder of the time, if he pays 3*l.* less than A?

iii. A, B, C divide their gains at the end of 12 months, and find them in the ratio of 2 : 3 : 4; their capitals were in the ratio of 4 : 3 : 2; what was the ratio of the time of their investments?

(13) i. A owes two debts of 200*l.* and 500*l.* respectively, which were to be paid in 6 months, but he pays the first in 3 months; when could he pay the other?

ii. A agrees to pay 1000*l.* by monthly instalments of 100*l.* each, but he pays the first five at once; when could he pay the remainder in one sum?

iii. A owes 4 Bills of 50*l.*, 80*l.*, 90*l.* and 150*l.* respectively; the first two he is to pay in 3 months, the third in 4 months, and the fourth in 6 months. He pays the first three bills at the equated time for paying them; when should he pay the last, and what is the equated time for the first three?

(14) i. Find the square root of 343·27953 to 8 figures, and of ·005 to 5 figures.

ii. Divide the square root of ·0076492516 by the cube root of 15625000.

iii. A cubical block of stone contains 234 c. ft. 1693·955125 inches; what is the area of the ground on which it stands?

(15) i. Exchange 825*l.* for marks, at 13 marks 12 schillings per 1*l.* (16 schillings making 1 mark).

ii. Exchange 550*l.* for marks at 13 marks 10½ schillings per 1*l.*

iii. If the par of exchange had been 13 marks 8 schillings per 1*l.*, then in (i.) how much per cent. premium was English money at Hamburg?

PAPER XV

(1) i. Find the greatest number that will divide 5148, 25038, and 897; also the least amount which can be exactly divided into heaps of the value of 7*d.*, 10*d.*, 1*s.*, 1*s.* 2*d.*, 1*s.* 6*d.*, 1*s.* 8*d.*, 2*s.*, 2*s.* 6*d.*, 2*s.* 11*d.* or 3*s.* 4*d.*

ii. Simplify $\dfrac{7\frac{2}{7}}{\frac{3}{34}}$ of $\dfrac{\frac{15}{19} \times \frac{13}{30}}{\frac{7}{38} \text{ of } 3\frac{1}{11}} \times \frac{1}{17}$ of 10 guineas.

iii. From the continued product of $2\frac{1}{24}$, $\frac{3}{5}$, $\dfrac{\frac{7}{12}}{\frac{3}{40}}$, $\frac{1}{7}$, $\frac{13}{14}$, and $\dfrac{2\frac{1}{4}}{22\frac{3}{4} \text{ of } \frac{1}{10}}$ subtract the difference between $3\frac{1}{7} + \frac{15}{28}$ and $2\frac{2}{21} + \frac{11}{14}$.

(2) i. A can hit the bull's eye 7 times out of 12, B 8 out of 15, C 9 out of 20, D 11 out of 24, E 17 out of 36, F 29 out of 45, and G 31 out of 48; which is the best shot? and arrange the others in order of merit.

ii. Add together $4\frac{2}{9}$, $3\frac{7}{15}$, $\frac{14}{27}$, $\frac{2}{3}$, $1\frac{1}{5}$, $2\frac{5}{9}$, and divide the sum by $\dfrac{14 \times \frac{11}{27} \times \frac{1}{12}}{\frac{13}{62} \text{ of } \frac{17}{57}} \times \dfrac{\frac{13}{38} \text{ of } \frac{14}{57}}{6\frac{10}{19} \times \frac{7}{51}}$.

iii. What decimal of 100*l.* is the sum of $\frac{19}{160}$ of 4*l.*, $\frac{5}{9}$ of 3 guineas, $\frac{7}{10}$ of 13*s.* 4*d.*, $\frac{5}{8}$ of a sovereign, and $1\frac{1}{8}$ of a shilling?

(3) i. The sum of two numbers is $8\frac{47}{75}$, and their difference $1\frac{37}{75}$; find the numbers, and express them as decimals.

ii. What number must be added to the difference between $17\frac{5}{18}$ and $13\frac{7}{24}$ to make it equal to the difference between the continued product of $\frac{7}{8}$, $\frac{13}{14}$, $\dfrac{2\frac{2}{11}}{\frac{1}{10}}$ and that of $1\frac{3}{4}$, $\frac{11}{17}$, $\frac{2}{11}$, $1\frac{3}{14}$? Express the result in both a fractional and decimal form.

iii. How much must be taken from the sum of $1\frac{1}{7}$ of a ton $\frac{5}{8}$ cwt. $3\frac{2}{7}$ qrs. $\frac{7}{16}$ lb. and $1\frac{13}{32}$ lb. to make it equal to the sum of $\frac{4}{7}$ of a ton $\frac{17}{28}$ of a cwt. $13\frac{11}{32}$ lbs. and $2\frac{5}{16}$ qrs.?

(4) i. Find the continued product of 583·2, 297 and ·000059; also of 27·34, 18·037, and ·00898.

ii. Divide ·694 by 175, and find what decimal multiplied by 11·2 will produce ·19847.

iii. Express in positive terms the difference between 2·5625 of a moidore + 2·36875 of 3⅔*l.* and 7·3125 of 1*l.* + 2·625 of 16*s.* 8*d.*

(5) i. What is the value of 529 tons 17¾ cwt. at 12*l.* 15*s.* 5¾*d.* per cwt.?

ii. Find the value of 127 qrs. 4 bus. 3 pecks of barley at 1*l.* 12*s.* 8*d.* per qr.

iii. Bought 3517 bales of cotton weighing 3 cwt. 15 lbs. each at 2*s.* 5*d.* per lb., and sold it at 2*s.* 1¾*d.*; find the loss per cwt., and also on the whole.

(6) i. How much tobacco was contained in 37 hhds., each 13 cwt. 14 lbs., allowing 16 lbs. per cwt. for tare?

ii. Find the net weight of 186 bags of rice, weighing 43 cwt. 2 qrs. 16 lbs., tare 20 lbs. per cwt., and tret 4 lbs. in 104 lbs.

iii. What is the net weight of 12 casks weighing 76 cwt. 1 qr. 7 lbs., tare 21 lbs. per cwt., tret 4 lbs. in 104 lbs., and draft 2 lbs. in 3 cwt.?

(7) i. What sum shall I receive as interest on 415*l.* 12*s.* for 9 yrs. 210 days at 6 per cent.?

ii. I bought 4 houses for 2500*l.* but could only pay down 1189*l.* 5*s.*, the rest remained on mortgage at 4 per cent. At the expiration of 8 years 175 days I paid the balance; how much had I paid altogether to the mortgagee?

iii. What sum lent at Compound Interest at 10 per cent. will amount to 1639*l.* 15*s.* $10\frac{2}{25}$*d.* in 4 years?

(8) i. Find the true present worth of 597*l.* 13*s.* 4*d.* payable in 8 months at 4½ per cent.

ii. What ready-money payment would discharge an account of 580*l.* 13*s.* 4*d.* due in 6½ months at 5 per cent.? and find the interest on 580*l.* 13*s.* 4*d.* for the same time at the same rate per cent.

iii. Find the difference between the bankers' and true discount of 1650*l.* 10*s.* 6*d.* for 9 months at 5 per cent.

(9) i. The 5 per cent. Stock is at 105, and the 4 per cent. Stock at 99$\frac{3}{4}$; in which shall I invest 500*l.* to produce the greatest income, and what will it be?

ii. A legacy of 2500*l.* is in 3 per cent. Stock, the value of which is 76$\frac{1}{2}$; the duty being 10 per cent., how much does it realise?

iii. A person has a certain sum in the 4 per cent. Stock, which is at 96, and he transfers it to the 5 per cents. at 125, thereby losing 20*l.* of income; how much 4 per cent. Stock had he?

(10) i. 20 cwts. of sugar are sold for 40*l.* 1*s.* 8*d.*, thereby gaining 10 per cent.; what did it cost per lb.?

ii. If an article when sold realised 5 per cent. profit on the cost, and the sale of $\frac{9}{10}$ of it gave a profit of 4*s.* 6*d.*, what did it cost?

iii. Bought tea to the value of 40*l.* 5*s.*, and had 6 months' credit. Sold the same at 5*s.* per lb., thereby gaining 5 per cent., and gave 3 months' credit; what was the gain upon the whole if interest be calculated at 5 per cent. per annum; and how many lbs. were there?

(11) i. A barters a quantity of rice with B for wheat at 8*s.* per bushel, the number of cwts. of rice being just $\frac{1}{3}$ the number of bushels of wheat, and the rice was worth 100*l.*; how many bushels of wheat were there, and what was the rice per cwt.?

ii. A charges B in barter 2*s.* 6*d.* per yard for linen which he would have sold at 2*s.* 4*d.* ready money. B buys 30*l.* worth of goods and pays 14*l.* down; how many yards does he buy?

iii. A agrees to sell a number of yards of linen to B in barter at 2*s.* 6*d.* per yard; it turns out better than was expected, and A cancels his bond by paying 2*l.* 5*s.* to B, which was 10 per cent. of the whole value, to be clear of his bargain; how many yards were bargained for?

(12) i. If A, B, C share 705*l.* in the ratio of $\frac{1}{3} : \frac{1}{4} : \frac{1}{5}$, what did they respectively receive?

ii. Suppose in (i.) 705*l.* was $\frac{1}{60}$ of their entire capital, what did they each invest?

iii. A and B engage 400 men and 100 men respectively on a railway, and B's men work longer than A's. The wages altogether come to 92*l.* 10*s.* per day and A pays $\frac{28}{37}$ of it; how much longer do B's men work a-day, and what are the wages of each?

(13) i. A owes three Bills of the respective values of 100*l.*, 200*l.*, 300*l.* He is to pay the first in 6 months, the second in 8 months, and the third in 10 months. If he can make 5 per cent. per annum of his money, find the equated time.

ii. A man borrows 500*l.* of Queen Anne's Bounty at 3½ per cent., to be paid off in 30 years, beginning with the second year; what will he pay in the tenth year?

iii. In (ii.) what would he lose by sinking 500*l.* instead of borrowing it, of Queen Anne's Bounty, reckoning interest of his money at 4½ per cent.?

(14) i. Find the area of a square whose side is 757·79 ft.; and the square root of ·000074685589241.

ii. Find the content of a cube whose edge is 23·9 inches in length; and the cube root of 29704593673.

iii. How much will it cost to paint the outside of a cubical box, whose capacity is 103 c. ft. 469·547 c. in., at 4¼*d.* per square foot?

(15) i. Exchange 3864 florins at Amsterdam with Naples at 80½ florins for 40 ducats.

ii. Bills bought in London on Paris at 25 francs 40 cents per 1*l.* are sold in Hamburg at 187 francs per 100 marks; what is the exchange between London and Hamburg?

iii. A Bill on Hamburg is bought at 13 marks 10¼ schillings per 1*l.*, and then sold at Amsterdam at 35¾ florins per 40 marks. The proceeds are afterwards remitted to Paris in French Bills at 57¼ florins per 120 francs; find the exchange between London and Paris.

PAPER XVI.

(1) i. Express $\frac{13}{5840}$, $\frac{113}{5931}$ and $\frac{290}{6153}$ with the denominators 99280, 421101 and 98448 respectively.

ii. Which is the greatest and which the least of the fractions $\frac{5}{7}$, $\frac{11}{14}$, $\frac{19}{35}$, $\frac{15}{28}$, $\frac{13}{21}$? and express the greatest as a decimal of the least.

iii. From what number must the sum of $\frac{3}{5}$, $\frac{7}{10}$, $11\frac{13}{15}$, $7\frac{11}{24}$, and $2\frac{17}{20}$ be taken so as to leave the quotient of $4\frac{5}{7}$ of $\frac{15}{99}$ of $\frac{13}{21}$ by $\frac{1}{12}$ of $\frac{8}{37}$ of $3\frac{24}{29}$ of $\frac{11}{56}$?

(2) i. Simplify $\left(\dfrac{13\frac{1}{5}}{4\frac{1}{2}} \times \dfrac{4\frac{1}{11} \text{ of } \frac{17}{18}}{1\frac{1}{9}}\right) + \left(4\frac{1}{4} \text{ of } \dfrac{\frac{5}{8}}{1\frac{5}{12} \text{ of } 3} \times \frac{9}{10}\right)$.

ii. Simplify $\left\{\left(2\frac{3}{5} \div 2\frac{4}{11}\right) + \left(7\frac{1}{8} - 3\frac{5}{12} + \frac{1}{2}\right)\right\} \div 7\frac{7}{12}$.

iii. Simplify $\dfrac{4\frac{3}{4} + \frac{7}{12}}{\frac{5}{8} \div 8\frac{1}{6} - 3\frac{7}{10}} \times \dfrac{\frac{7}{8} \text{ of } \frac{5}{9} \text{ of } 3}{1\frac{5}{6} + 3\frac{11}{18}}$.

(3) i. What number must be subtracted from the difference between 2729 tenths + 13 + 13 thousandths and 2·2164 + 18 + 9 hundredths to make it equal to the difference between 77·9374 and 65·3216853?

ii. To the continued product of 23·46, ·0347, and ·00594 add the quotient of ·73469 by ·0625.

iii. Express in positive terms 10·275 of a sovereign + ·0825 of 5 guineas + 27·25 of 1*l.* 13*s.* 6*d.*

(4) i. Express as vulgar fractions $1{\cdot}29\dot{1}4285\dot{7}$; ${\cdot}37\dot{4}1\dot{2}$; ${\cdot}003\dot{7}\dot{8}$.

ii. Express as decimals $17\frac{51}{875}$; $\frac{149}{15625}$; $\dfrac{4\frac{3}{4}}{6\frac{1}{3}} + \dfrac{5{\cdot}5 - {\cdot}75}{2\frac{5}{7}}$.

iii. Simplify $\dfrac{1{\cdot}3\dot{6} \times {\cdot}4\dot{8}5714\dot{2}}{{\cdot}22\dot{7}} \times \dfrac{{\cdot}\dot{9}5238\dot{0}}{\frac{1}{12} \text{ of } 1{\cdot}4} \times \dfrac{\frac{7}{15}}{\frac{34}{49}} \text{ of } \frac{5}{7}$.

(5) i. What must be given for $479\frac{5}{9}$ cwt. of raisins at 5*l.* 14*s.* $8\frac{1}{4}$*d.* per cwt.?

ii. Bought a piece of cloth measuring 97 yds. 2 qrs. 1 nl. at 1*l.* 7*s.* 4*d.* per yard, and another measuring 74 English ells 4 qrs. 2 nls. at 15*s.* 10*d.* per ell; find the total cost.

iii. Find the value of 30 tons 2 cwt. 1 qr. 23 lbs. at 13*s.* 8*d.* per lb.

(6) i. What is the net weight of 73 boxes of soap weighing 429 cwt. 3 qrs. 7½ lbs., tare 15½ lbs. per box, tret 4 lbs. in 104 lbs.?

ii. Find the net value of 81 casks of coffee, each 2 cwt. 3 qrs. 16 lbs., tare on the whole 27 cwt. 2 qrs. 15 lbs., at 7*l.* 11*s.* 8*d.* per cwt.

iii. Find the net weight of 70 hhds. weighing 315 cwt. 2 qrs. 9 lbs., whole tare 31 cwt. 1 qr. 6 lbs., tret 4 lbs. in 104 lbs., and draft 2 lbs. in 3 cwt.

(7) i. What sum lent at Simple Interest at 7⅕ per cent. will amount to 972*l.* 13*s.* $9\frac{3}{10}$*d.* in 2 yrs. 11 months?

ii. In what time will the interest on 224*l.* 13*s.* 9*d.* become 42*l.* 13*s.* 9¾*d.* at 4 per cent.?

iii. Compound Interest of 465*l.* 12*s.* 6*d.* for 4 yrs. at 6¼ per cent.

(8) i. Find the present worth of 900*l.* 17*s.* 6*d.* due in 21 months at 3⅞ per cent.

ii. How much would a banker deduct more than his due from a Bill of 386*l.* 9*s.* 8¼*d.* due in 7 months at 5¼ per cent.?

iii. What will a banker gain by discounting a Bill of 565*l.* 15*s.* drawn July 24th at 8 months, and discounted October 3rd at 4 per cent.?

(9) i. How much Stock in the 4¾ per cents. will produce half the income of 500*l.* in the 3⅜ per cent. Stock?

ii. Bought shares to the amount of 459*l.* at the rate of 76½ per cent. and sold them at 3½ per cent. premium. If they were 10*l.* shares, how many did I buy, and what did I sell them for?

iii. Bought 10*l.* shares of the 6 per cent. preference Stock, with 6*l.* paid up, at 6*s.* 8*d.* discount. If the half-yearly dividend on the money paid up came to 27*l.*, how many shares did I buy, and what rate of interest did I get for my investment?

(10) i. Mixed tea at 4*s.* 7½*d.* per lb. with tea at 5*s.* 3½*d.* per lb., and sold the mixture at 4*s.* 10½*d.* per lb.; how did I mix them so as neither to gain nor lose?

ii. A person goes to market with 12*l.*, with which he wants to buy tea, sugar, and coffee. If he pays 1*s.* 6*d.* per lb. for the last, and spends 6*l.* in tea, which is 50 per cent. more than the cost of the sugar at 4*d.* per lb., how much sugar and coffee does he buy?

iii. A grocer mixed tea at 4*s.* 5*d.*, 4*s.* 9*d.*, and 6*s.* 9*d.* per lb. respectively, and sold the mixture at 5*s.* per lb.; how did he mix them?

(11) i. A mixes tea at 4*s.* per lb. with some at 6*s.* per lb., and sells the mixture to B, in barter, at 5*s.* per lb. for cheese, a cwt. of which is equal to 10 lbs. of the mixture. How many lbs. of each sort of tea will A give for 20 cwt. of cheese?

ii. Supposing in (i.) A had sold the mixture at 4*s.* 6*d.* per lb., and B's cheese had been worth 30*l.*, how much of each sort of tea would he have mixed?

iii. Suppose in (ii.) A's tea had proved inferior, and B's cheese superior, to the sample, and A paid 1*l.* 16*s.* to B to make up the difference, what ought to have been the price of the mixture, supposing the same quantity?

(12) i. A, B, C trade and gain 105*l.*, which they share in the ratio of $\frac{1}{2}$: $\frac{1}{4}$: $\frac{1}{8}$; how much had each?

ii. In (i.) the whole capital was 1260*l.*; how much per cent. of their capitals did they gain?

iii. If A, B, C gain $14\frac{2}{7}$ per cent. of their capital, and A's gain was 210*l.*, what had B and C, if their capitals were in the proportion of 2 : 3?

(13) i. If 100*l.* be due now and 50*l.* at the end of each month for 8 months, find the equated time.

ii. $\frac{1}{3}$ of a debt of 400*l.* is to be paid in 3 months, $\frac{1}{5}$ in 5 months, $\frac{1}{6}$ in 6 months, and the rest in 10 months; find the equated time.

iii. If in (ii.) 400*l.* were paid in 4 months to commute the debt, how much was lost at 5 per cent. per annum?

(14) i. Add together the square of 53·49 and the square root of 865093·26479521.

ii. Find the square root of 293·21726538 to four figures, and prove it.

iii. Extract the cube root of 19759·644267875.

(15) i. If a franc is worth $9\frac{3}{5}$*d.*, and a man has an income of 470*l.*, how many francs will he get in Paris in exchange?

ii. Suppose in (i.) English bank notes at a premium of 4 per cent. in Paris, what would he get in exchange if his money had been remitted in English notes?

iii. If in (i.) he received 12,000 francs and wished to exchange it into English money when at a discount of 5 per cent., how much would he get?

PAPER XVII.

(1) i. From $2\frac{1}{15}$ subtract $1\frac{19}{35}$ and divide the result by the difference between $\frac{11}{12}$ and $\frac{13}{84}$.

ii. What fraction added to the continued product of $\frac{31}{80}$, $2\frac{1}{7}$, $\frac{4}{93}$, and $2\frac{6}{11}$ will give the continued product of $\frac{7}{8}$, 9, $\frac{2}{3}$, $1\frac{11}{21}$, and $\frac{3}{4}$?

iii. A gives $\frac{3}{5}$ of $\frac{20}{21}$ of $\frac{5}{9}$ of $\frac{9}{10}$ of his money to B, $\frac{9}{10}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{2}{3}$ of what was left to C, and $\frac{9}{11}$ of $\frac{7}{8}$ of $\frac{11}{14}$ of $\frac{8}{9}$ of the remainder to D; he had still 10 guineas remaining; how much had A at first? and find the share of each.

(2) i. The sum of two numbers is $9\frac{201}{224}$, the greater is $5\frac{31}{36}$; find the other, and divide it by $\dfrac{\frac{3}{4} \text{ of } \frac{5}{6}}{1\frac{1}{2}}$.

ii. From the sum of $4\frac{3}{7}$, $2\frac{1}{14}$, $\frac{11}{12}$, $\frac{17}{21}$, and $4\frac{11}{28}$ take the sum of $\frac{9}{10}$, $\frac{11}{20}$, $\frac{13}{15}$, $4\frac{7}{30}$, $\frac{11}{18}$, and $\frac{5}{36}$.

iii. Divide 3*l.* 17*s.* $10\frac{1}{2}$*d.* by $\frac{623}{640}$; 1*l.* 17*s.* $10\frac{1}{2}$*d.* by $3\frac{17}{28}$; 7*s.* $6\frac{3}{4}$*d.* by $\frac{11}{100}$, and reduce the second to the fraction of the sum of the first and last.

(3) i. Express $7\frac{1}{4}$ of $11\frac{5}{84}$ as a decimal, and 13·$\dot{8}$1467$\dot{1}$ as a vulgar fraction.

ii. Multiply 13·954 by ·00087, and divide ·3964687 by 156·25.

iii. Divide 1*l.* 13*s.* 10½*d.* by ·6775; multiply 3*l.* 13*s.* 4*d.* by 2·36875; subtract 2·5625 of a moidore from 7·3125 of 1*l.*, and find the excess of the second result over the sum of the other two.

(4) i. Find the sum of $27{\cdot}\dot{3}$, $\cdot 1\dot{6}$, $39{\cdot}\dot{2}\dot{2}$, $3{\cdot}\dot{1}4285\dot{7}$.

ii. Find the sum of the figures in the circulating decimals equivalent to $\frac{52}{875}$, $\frac{23}{1008}$, and $\frac{5}{52}$.

iii. Simplify—

$$\frac{\cdot 8247}{\frac{4}{33}\text{ of }2{\cdot}625}+\frac{20\frac{1}{8}}{\cdot 7291\dot{6}}+\frac{1{\cdot}\dot{3}\times 4{\cdot}75\times 3{\cdot}\dot{1}4285\dot{7}\times\frac{3}{4}}{8{\cdot}\dot{1}4285\dot{7}\times 1\frac{1}{15}\times 1{\cdot}8\dot{3}\times\cdot 46875}-\frac{\frac{5}{7}\text{ of }\cdot 6\text{ of }2\frac{2}{3}}{4{\cdot}5\times\cdot\dot{2}9\dot{6}\times\frac{12}{35}}.$$

(5) i. Find the value of 735 barrels of oil, each containing 18 gallons, at 6*s.* 10¾*d.* per gallon.

ii. Find the value of 27 boxes, each containing 16 ingots of gold, at 3*l.* 10*s.* per oz., each ingot weighing 5 lbs. 6 oz. 8 dwts.

iii. Find the cost of repairing two roads, one of which measures 84 miles 6 fur. 20 poles, at 12*s.* 4*d.* per mile; and the other, 59 miles 6 fur. 37 poles, at 1*l.* 15*s.* 6*d.* per mile.

(6) i. Find the net value of 23 boxes of linseed, each 2 cwt. 3 qrs. 15 lbs., tare per box 22 lbs., at 4½*d.* per lb.

ii. Find the net weight of 27 barrels weighing 118 cwt. 3 qrs. 12 lbs., tare 17½ lbs. per cwt., tret 4 lbs. in 104 lbs.

iii. Find the net weight of 64 hhds. each 10 cwt. 3 qrs. 24½ lbs., tare 12⅘ lbs. per cwt., tret 4 lbs. in 104 lbs., and draft 2 lbs. in 3 cwt.; and what would they amount to at 11*l.* 13*s.* 4*d.* per cwt.?

(7) i. What sum shall I receive as interest on 634*l.* 12*s.* 6*d.* for 7 yrs. 195 days, at $6\frac{7}{11}$ per cent.?

ii. Find the Compound Interest of 734*l.* 7*s.* 6*d.* for 3½ years at 4 per cent.

iii. What sum lent at Compound Interest at 6¼ per cent. will amount to 1427*l.* 7*s.* $2\frac{77}{128}$*d.* in 4 years?

(8) i. Find the discount and present worth of a Bill of 117*l.* 18*s.* 1½*d.*, due in 20 months, at 6 per cent.

ii. Find the discount of 32*l.* 5*s.* 10¾*d.*, due in 5 months, at 7⅕ per cent., and also the interest of the present worth for the same time, at the same rate per cent.

iii. On the 3rd of December 1863 I took a Bill of 4036*l.* 5*s.* 10*d.* to the Bank to be discounted. It was drawn October 18th of the same year for 10 months; what did the banker give me, the rate of discount being 6¼ per cent.?

(9) i. Bought 2400*l.* 4 per Cent. Stock at 96, including the broker's commission of ⅛ per cent.; how much money did I invest?

ii. What amount of Stock in the 3½ per Cents. will produce the same income as 2316*l.* 13*s.* in the 3 per Cents.?

iii. How many shares in the Manchester 10*l.* Preference Stock, with 6*l.* paid up and guaranteed 6 per cent. interest, will be equivalent to as much of the ordinary Stock as pays an income of 36*l.* 6*s.* 8*d.*?

(10) i. Sold tea at 4*s.* 6*d.* per lb. and also at 6*s.* per lb., and mixed them in the proportion of 4 of the first to 3 of the second. Suppose I charged them, before mixture, at a profit of 5 per cent., what should I gain per cent. altogether by selling the mixture at 5*s.* 3*d.* per lb.?

ii. In (i.) if I had mixed the first at 4*s.* 1³⁄₇*d.* per lb., with the second at 5*s.* 2⁴⁄₇*d.*, and an inferior tea at 5*s.*, on which I gained 30 per cent., in the proportion of 1 lb. of this to 2 lbs. each of those, and sold the mixture at 4*s.* 10*d.* per lb., what should I have gained or lost per cent. on the original cost?

iii. Sold an article at a profit of 10 per cent., but, owing to the failure of the person who bought it, I only received 21*l.*, being an instalment at the rate of 17*s.* 6*d.* per 1*l.*; what did I give for it, and how much per cent. did I lose?

(11) i. A buys 1152 English ells of cloth at 2*s.* 6*d.* per English ell, and retails the same at 3*s.* 6*d.* in barter to B; how many yards could B buy by paying 70*l.* cash, and giving

silk at 3*s.* 6*d.* per yard; and how many yards of silk would he give?

ii. A mixes a quantity of tea at prices of 3*s.*, 4*s.*, and 6*s.* per lb. respectively; what quantity does he mix of each, to barter it with B at 4*s.* per lb. for sugar which came to 54*l.*?

iii. Suppose in (ii.) he had mixed the tea in the ratio of 2 : 3 : 4 lbs. respectively; what would he have lost or gained per cent. upon the former transaction, and what must B have charged for his sugar to make it an equivalent bargain?

(12) i. A, B, C rent pasture for 10 months, at 4*s.* 6*d.* a head for cows per month; they put in cows in the proportion of 1 : 2 : 3, but at the end of 6 months A sells his cows to B and C in the proportion of 1 : 3. If the rent came to 108*l.*, how much did each pay?

ii. A and B pay 200*l.* income-tax at 6*d.* in the 1*l.*, and their portions are as 4 : 7; what was A's capital if they gained 5 per cent.?

iii. 150*l.* is distributed to 10 men and 10 women, so that the men and women have shares in the ratio of 2 : 3; how much per cent. of the whole does each receive?

(13) i. Three sums of 250*l.* 3*s.* 4*d.*, 500*l.* 6*s.* 8*d.*, and 600*l.* 8*s.* 4*d.* respectively, are severally due in 2, 3, and 4 months. For the first two a Bill of 750*l.* 10*s.* is given at once, which was 3 months overdue, and bore interest at 4 per cent.; when must the third sum be paid, reckoning interest at 5 per cent. for the three sums?

ii. What would a person lose by paying 100*l.* cash, if he could have 10 years to pay it in, by equal instalments, at 4 per cent. per annum, supposing he could make 5 per cent. of his money?

iii. Two sums of 160*l.* 5*s.* and 240*l.* 15*s.* respectively were due two months ago; what sum must be paid in 4 months to commute them, reckoning interest at 4½ per cent. per annum?

(14) i. Extract the square root of ·0000016359177409 and of 65130645·966736.

ii. How many yards of fencing are required for a square field containing 738 acres 502 yds. 7 ft. 135·455625 inches?

iii. A cubical tank holds 201659 c. ft. 1047·653 c. in. of water, and a walk goes round three sides of it; find the length of this walk in yards.

(15) i. Exchange 550 rupees, at $23\frac{1}{2}$*d.* each.

ii. If English money is at a discount of 3 per cent., what will 970 rupees realise?

iii. A banker sells 250 sovereigns at 25·30 francs per 1*l.*, and buys marks at 13 marks 8 schillings per 25·15 francs. He then sells a mark at 1·8 francs, and buys sovereigns when English money is at a discount of 5 per cent. in Paris, and the par value of a franc is $9\frac{2}{5}$*d.*; does he gain or lose?

PAPER XVIII.

(1) i. Find the highest common divisor of 5544, 4248, and 12492; also the least common multiple of 7, 10, 15, 18, 20, 22, 24, 30.

ii. Find the greatest number common to 16502, 9546, 4588, and 145299; and reduce $347\frac{4}{93}$ and $43\frac{27}{527}$ to improper fractions.

iii. Express in their simplest form $\frac{3744}{1123248}$; $(7\frac{3}{4}+8\frac{1}{11}+1\frac{15}{16})-(1\frac{1}{2}+3\frac{2}{3}+1\frac{1}{4}+\frac{2}{3})$.

(2) i. Bought $\dfrac{\frac{5}{8}\text{ of }\frac{24}{25}}{1\frac{1}{100}\text{ of }\frac{8}{49}}$ of $\frac{303}{500}$ of $\dfrac{\frac{27}{50}}{3\frac{1}{16}}$ of an estate for 1828*l.* 11*s.* 6*d.*; what did the whole cost?

ii. To what number must the continued product of $\frac{7}{10}$, $\frac{11}{21}$ of $\dfrac{\frac{30}{51}\text{ of }7\frac{5}{7}\text{ of }\frac{4}{9}}{1\frac{3}{17}\text{ of }\frac{3}{32}\text{ of }3\frac{1}{5}}$ be added, so as to make it equal to the difference between $17\frac{11}{18}$ and $\frac{39}{40}$?

iii. Simplify $\dfrac{1\frac{1}{3}\text{ of }3\cdot75\text{ of }\cdot5}{2\cdot\dot{5}7142\dot{8}-\frac{2}{3}\text{ of }\frac{6}{7}}-\dfrac{3\cdot\dot{3}+1\cdot0\dot{6}}{1\frac{1}{9}+\cdot\dot{5}7142\dot{8}+3\cdot\dot{3}1746\dot{0}}$, expressing the result in both a fractional and decimal form.

(3) i. What is the difference between $4\frac{1}{4}$ of $\frac{3}{8}$ of $\frac{16}{17}$ of 20 of $\frac{1}{5}$ of 5*l.* 2*s.* 6*d.* and $\frac{3}{8}$ of $\frac{7}{10}$ of $\frac{5}{3}$ of 6 of $\frac{7}{20}$ of 13*l.* 6*s.* 8*d.*?

ii. Find the value of £$7\frac{11}{120}$ + $\frac{13}{81}$ of a guinea + $\frac{5}{9}$ of a moidore + $\frac{1}{320}$ of a sovereign + $\frac{19}{480}$ of 10*l.*

iii. From the sum of £$15\frac{5}{32}$ and $\frac{37}{84}$ of a guinea take the sum of $\frac{3}{7}$ of 7*l.* 10*s.* $9\frac{1}{2}$*d.*, $\frac{19}{81}$ of a moidore, and $1\frac{5}{6}$ of 6*d.*

(4) i. What decimal multiplied by the product of ·0349 and ·00785 will give ·00000204651855?

ii. The sum of two numbers is $34{\cdot}84\dot{0}\dot{9}$ and their difference $4{\cdot}06\dot{8}1$; find the two numbers.

iii. Express in positive terms ·875 cwt. + 7·03125 of a ton + 3·0078125 of 1 ton 6 cwt.

(5) i. How much must be paid for 26 weeks' wages of 315 labourers, averaging 2*s.* $6\frac{1}{2}$*d.* per day?

ii. Find the dividend on 65937*l.* 16*s.* 8*d.* at 13*s.* $5\frac{3}{4}$*d.* in the £.

iii. Find the freight on two shipments, one of 26 tons 16 cwt. 3 qrs. 18 lbs. at 18*s.* 8*d.* per cwt., and the other 19 cwt. 2 qrs. 25 lbs. at 26*l.* per ton.

(6) i. Find the net weight of 130 boxes, each 2 cwt. 3 qrs. 14 lbs., tare 16 lbs. per box, and tret 4 lbs. in 104 lbs.

ii. Find the net value of 32 bales of cotton, each 1 cwt. 3 qrs. 12 lbs., tare $10\frac{1}{2}$ lbs. per cwt., tret 4 lbs. in 104 lbs., at 1*s.* $8\frac{1}{4}$*d.* per lb.

iii. Bought 129 chests of tea, each 1 cwt. 3 qrs. 16 lbs.; find the net weight, allowing 4 cwt. 3 qrs. 20 lbs. for the weight of the chests, 4 lbs. in 104 lbs. for tret, and 2 lbs. in 3 cwt. for draft.

(7) i. How much will 400*l.* 4*s.* $2\frac{50}{301}$*d.* amount to at Simple Interest in 12 yrs. $7\frac{1}{2}$ months, at 4 per cent.?

ii. In what time will 888*l.* 9*s.* $3\frac{1}{4}$*d.* amount to 1350*l.* 9*s.* $3\frac{1}{2}$*d.*, at $2\frac{3}{5}$ per cent.?

iii. Compound Interest of 912*l.* 10*s.* for 3 years, at $3\frac{3}{4}$ per cent.

(8) i. Find the discount of 256*l.* 2*s.* $1\frac{1}{4}$*d.* for 3 years 5 months, at 4 per cent.

ii. How much does a banker receive more than his due who discounts at 7½ per cent. a Bill of 1056*l.* which has 5 months to run?

iii. Find the present value of a debt of 786*l.* 0*s.* 10*d.*, half of which is payable in 4 months, and the other half in 8 months, at 6¼ per cent.

(9) i. Bought 4 per Cent. Stock at 95, and realised an income of 95*l.* per annum; how much money did I invest?

ii. A person had 15 shares of Dutch-Rhenish 20*l.* Stock, with 16*l.* paid up; also 1300*l.* Magnetic Telegraph Stock. If the first-named sells at 5⅜*l.* premium, and the latter at 28½ per cent. discount, how much will they realise if sold out?

iii. In (ii.) how much London and North-Western Stock at 122½ can be bought by the Dutch-Rhenish and Magnetic Stocks? If the income be 30*l.* for the half year, what is the dividend on the London and North-Western Stock?

(10) i. Sold a cow for 15*l.*, thereby losing 10 per cent., but I intended to have sold it at 10 per cent. profit; how much less than the price I had fixed upon did I sell it for?

ii. A lb. of mould candles costs 8*d.* and contains 6 to the lb.; a lb. of dips costs 6½*d.* and contains 10 candles. One mould burns as long as two dips, i.e. 6 hours; how much per cent. cheaper are dips than moulds?

iii. Bought sugar to the value of 20*l.*, at 2*l.* per cwt. It was kept 6 months in bond, and was found to have been damaged by the sea. The sound part was sold at 20 per cent. less than cost, and realised 12*l.* If the interest of money was 4 per cent. per annum, what would be the entire loss, and how much was damaged?

(11) i. A owes B 150*l.*, which they settle by means of an interchange of goods. A has wool which would realise 34*l.* more than the debt, and B takes the whole of this, giving him tea to make up the balance, charging in barter 5 per cent. more than his ready-money price,

which was 4*s.* per lb. For what does B sell his tea per lb., and how much was there?

ii. If in (i.) the wool had been 2*s.* 6*d.* per lb. in barter, and B had agreed to take 30 per cent. of the debt in wool, at a ready-money price proportionate to the ready-money price of the tea, and the rest in money; how much wool would A have had left, and how much money would A have had to pay?

iii. A has cotton at 2*s.* per lb., B has wool, and C stockings at 2*s.* per pair. A owes B 100*l.* for wool, and C owes A for cotton. If C sold B 6000 pairs of stockings at 10 per cent. less than his fixed price, on condition that B should pay A $\frac{2}{5}$ of C's debt; what was C's debt to A, and how much cotton did he buy?

(12) i. A, B, and C join in a transaction. A's gain was calculated at 1000*l.*, B's at 500*l.*, and C's at 300*l.*, but the whole gain was found to amount only to 1250*l.*; how did they share it?

ii. A and B hire a field for 60*l.* a year. A puts in 6 cows, 4 horses, and 24 sheep for 6 months; B puts in 2 horses and 40 sheep for 6 months, and afterwards takes out these, putting in cows for the remainder of the time, and pays the rest of the rent; how many cows had B, if two cows are equal to a horse, and 4 sheep equal to a cow, and what did each pay?

iii. A and B invest 2000*l.* in a company, the shares of which pay 5 per cent., and are 20*l.* each. At the price at which they bought the shares the money invested paid 6 per cent.; how many shares had each, if A receives 90*l.* interest?

(13) i. If 100*l.* is to be paid in 30 equal annual instalments, bearing interest at $3\frac{1}{2}$ per cent.; find the twentieth payment.

ii. Three debts of 200*l.*, 300*l.*, and 400*l.* respectively, are severally due in 3, 4, and 5 months. These were commuted by one payment in 4 months; what was it, reckoning interest at 5 per cent.?

iii. In (ii.) if the first debt had been 3 months overdue, what sum would have commuted the whole in 4 months?

(14) i. Find the square root of 49613639253025, and the number of square yards in a field whose length and breadth are each 5349 ft.

ii. Find the cube root of ·26 to four figures, and the length of the side of a cubical mound containing 178453·547 c. inches.

iii. What is the least number which when subtracted from 128596450932893 will make it an exact cube?

(15) i. Exchange 3225 American dollars for English money when it is at a premium of $7\frac{1}{2}$ per cent.

ii. A Bill of 200*l.* was drawn in London, and paid in Paris at an exchange of 25·15 francs per 1*l.* A Bill for the amount was drawn in Paris on India for rupees, at 94 francs for 40 rupees; what did it realise in India?

iii. Suppose in (ii.) the Bill returned from India to England, at an exchange of $23\frac{1}{2}$*d.* per rupee, would it be a gain or loss upon the direct route?

PAPER XIX.

(1) Simplify the following expressions, and give the result in a fractional as well as a decimal form:—

i. $17\frac{3}{4}+17\frac{1}{2}+\frac{7}{20}+\frac{11}{28}+8\frac{7}{10}+9\frac{19}{56}+4\frac{5}{8}$.

ii. $\dfrac{7\frac{3}{10}}{\frac{1}{9}}\times\frac{17}{16}\times\dfrac{\frac{33}{34}}{13\frac{14}{27}}\times\dfrac{20\times11\times96}{21\times12\times5}\times\frac{5}{12}\text{ of }\frac{7}{11}\text{ of }\frac{1}{9}\text{ of }\frac{3}{8}\text{ of }\dfrac{\frac{5}{8}}{1\frac{1}{5}}$.

iii. $17\frac{3}{5}-\frac{11}{20}-1\frac{17}{18}+\frac{14}{15}-2\frac{13}{40}+11\frac{5}{6}-\frac{7}{16}-\frac{2}{3}$.

(2) i. Which is the greatest and which the least of the fractions $\frac{6}{11}$, $\frac{3}{11}$ of $2\frac{1}{3}$, $2\frac{1}{4}$ of $\frac{3}{11}$, $\frac{5}{11}$ of $1\frac{1}{6}$, and $\frac{17}{33}$, and find the sum of the next to the greatest and the next to the least.

ii. What number must be multiplied by $\dfrac{\frac{7}{11}}{\frac{6}{13}\text{ of }6\frac{1}{3}}\text{ of }\dfrac{1\frac{4}{11}\text{ of }\frac{15}{26}}{\frac{75}{78}}$ to make it equal to the difference between the sum of $\frac{15}{28}$ and $3\frac{1}{7}$, and the sum of $2\frac{2}{21}$ and $\frac{11}{14}$?

iii. Simplify $\frac{5}{8}$ of a ton + $\frac{3}{16}$ of a cwt. + $\frac{11}{14}$ of a quarter + $\frac{7}{32}$ of a lb. + $\frac{11}{12}$ of 3 tons.

Simplify the following expressions:—

(3) i. 583·7 + 9 hundredths + 58379 − 583 thousandths − 58 − 79 hundred-thousandths − 379 thousandths + 387·64 − 3752 tenths − 12 thousandths − 59·5 − 78689 thousandths + 49 thousandths + 372 tenths − 3834 − 7264 ten-thousandths − 869 ten-millionths − 9 − 753 ten-thousand-millionths.

ii. ·1574 × ·01057 × 864 hundred-thousandths; ·896 × 289 × ·00896.

iii. 783·6 + 8·75; 5846 ÷ ·003125; ·07234 + ·00015625.

(4) i. Suppose a train goes 30 yds. the first second after the steam is shut off, $\frac{3}{4}$ of that distance in the second second, $\frac{1}{2}$ of it in the third, $\frac{1}{3}$ of it in the fourth, $\frac{1}{4}$ in the fifth, $\frac{1}{5}$ in the sixth, $\frac{1}{6}$ in the seventh, and $\frac{1}{7}$ in the eighth, when it is brought to a standstill; find the distance gone after shutting off the steam.

ii. Find the salary of a cashier who has 834*l.* 17*s.* 11*d.* remaining after paying an income-tax of $\cdot 91\dot{6}$ of a shilling per 1*l.*

iii. Simplify
$$\left(\frac{33\cdot0208\dot{3}}{6\cdot6041\dot{6}}+\frac{11\cdot\dot{0}\dot{9}}{2\cdot7\dot{7}\dot{2}}\right)+\frac{\cdot006235}{\cdot00\dot{2}5}-\left(\frac{2\cdot625}{\cdot\dot{7}}-\frac{5\frac{1}{9}\text{ of }\frac{18}{23}\text{ of }3}{\frac{3}{4}\text{ of }16\div\frac{1}{3}\text{ of }4\frac{1}{2}}\right)$$

(5) i. A person owes 5955*l.* 17*s.* 6*d.*; how much of this can he pay at 10*s.* 6½*d.* in the £1? And what loss do the creditors sustain?

ii. Find the value of 25 hhds. of coffee, weighing net 156 cwt. 2 qrs. 17 lbs. at 1*l.* 10*s.* 8*d.* per cwt.

iii. Out of 9 hhds. 2 gals. of wine, 3 hhds. 3 gals. 1 pt. were sold; find the value of what was left at 1*l.* 9*s.* 9*d.* per gallon.

(6) i. Find the net value of 127 firkins of butter, weighing gross 10 tons 6 cwt. 16 lbs. at 1*s.* 2½*d.* per lb., tare 18 lbs. per firkin.

ii. Find the net weight of 8 hhds. weighing gross 93 cwt. 21 lbs., tare 14 lbs. per cwt. and tret 4 lbs. in 104 lbs.

iii. Find the number of boxes, each holding 12½ lbs. that can be filled from 1400 chests of tea, weighing gross 1327 cwt. 2 qrs., allowing 17½ lbs. per cwt. for tare, 4 lbs. in 100 lbs. for tret, and 2 lbs. in 252 lbs. for draft.

(7) i. What principal will amount to 527*l.* 2*s.* 0½*d.* in 7 yrs. $3\frac{3}{7}$ months, at 6 per cent.?

ii. At what rate per cent. will the interest of 713*l.* 17*s.* become 322*l.* 12*s.* $1\frac{7}{88}$*d.* in 4 yrs. 27 weeks?

iii. Find the difference between the Simple and Compound Interest of 873*l.* 15*s.* for 4 years, at 6¼ per cent.

(8) i. What is the discount of 74*l.* 14*s.* 2½*d.* due in 1 yr. 5 months, at 4¾ per cent.?

ii. What will a banker gain by discounting a Bill of 1432*l.* 4*s.* 10¾*d.* drawn October 12, at 9 months, and discounted January 1, at 6 per cent.?

iii. Find the present value of a Bill of 1796*l.* 8*s.* 9*d.*, ⅓ of which is due in 3 months, ⅓ in 6 months, and the rest in 9 months, at 6¼ per cent.

(9) i. What rate of interest arises from money invested in the 3 per Cents. at 84½?

ii. Transferred from the Danish 5 per cent. Stock, at 105, to the Dutch 4 per cent. Stock at 88⅗, and lost 10*l.* of income; how much Danish Stock had I?

iii. Bought 5 per cent. Stock at 10 per cent. premium, and sold it for 4¾ per cent. Stock which was at 5 per cent. premium. Supposing I lost 20*l.* in income, how much of the first Stock had I?

(10) i. Cloth when sold at 14*s.* 3*d.* per yard realises a profit of 14 per cent.; at what price per yard should it be sold to clear 20 per cent.?

ii. If 14 lbs of tea at 3*s.* 6*d.* per lb. be mixed with 8 lbs. of another sort, and the mixture sold for 4*s.* a lb., find the price of the second kind.

iii. In what proportion should tea be mixed to sell the mixture at a profit of 8 per cent. on the prime cost, supposing the prices 3*s.* 4*d.* and 5*s.* per lb., and these to be a gain of 5 per cent. on their prime cost already?

(11) i. How many yards of cloth at 12*s.* 6*d.* per yd., and 10*l.* in cash, must A give B for 200 yds. of linen at 3*s.* 6*d.* a yard?

ii. If in (i.) A's cloth had been 15 per cent. dearer, and the same quantity, how much money would he have had to give B?

iii. If A's goods are 12 per cent. dearer than B's, and A has 200 yards of cloth; how many yards of linen must B give in barter for them?

(12) i. A and B share 4310 shillings, in the ratio of $3\cdot\dot{7}$*l.* : 4·15 guineas; what had each?

ii. If A's share of the gain in a transaction were $\left.\right\} \frac{23}{1\cdot7} \times \frac{13\cdot85}{1\cdot02} \times \frac{1\cdot21}{4\cdot9}\, l.$, what was B's if it were 10 per cent. more?

iii. A gains $3\frac{1}{5}$*l.* per cent. per month, and B 30 per cent. per annum. They trade for 12 months, and B's gain is 50*l.*; find the capital, if B's be $\frac{2}{3}$ of A's; also A's gain.

(13) i. A debt of 105*l.* 6*s.* is paid in monthly instalments of 8*l.* 15*s.* 6*d.*; the interest is $\frac{1}{2}$ per cent. on the first payment, $\frac{1}{3}$ per cent. on the second, and so on; find the 9th monthly payment.

ii. 700*l.* is paid thus: $\frac{1}{3}$ cash; of the remainder, $\frac{1}{6}$ in 4 months, $\frac{1}{4}$ in 5 months, $\frac{1}{2}$ in 7 months, and the rest in 9 months; find the equated time.

iii. If I pay 252*l.* 18*s.* 4*d.* in $3\frac{3}{5}$ months to commute two payments of 100*l.* and 150*l.* respectively, the first of which was due in 3 months, when was the second due?

(14) i. Find the square root of ·00021813222249, and the number of acres in a square field the distance round which measures 1 mile 379·6 yards.

ii. The solidity of a cubical block of wood is 7890 c. ft. 869·869 c. in.; find the area of its surface.

iii. Find the cube root of 26 to four figures; also extract the cube root of $\frac{185193}{493039}$.

(15) i. If exchange with London and Paris is 1*l.* per $25\frac{1}{2}$ francs, with Paris and India 40 rupees for 94 francs, and if English money is at a premium of 5 per cent. in India, will it be better to send 100*l.* by this route, or direct, the ordinary exchange being $23\frac{1}{2}$*d.* per rupee at par?

ii. If the exchange at par with London and Paris is 1*l.* per 25·30 francs, but 25·50 francs are obtained, is English money at a premium or discount in Paris, and how much per cent.?

iii. The premium on gold in Paris is $5\frac{1}{2}$ per mille, which gives an exchange of 25·29 francs per £1; if gold is 26 per cent. dearer in Paris than in London, then show that the exchange in Paris on London is about 25·225 francs per £1.

PAPER XX.

(1) i. What mixed numbers are respectively equivalent to $\frac{4318529}{1208}$; $\frac{73469848}{2048}$; $\frac{346275375}{31625}$?

ii. Find the least number of pins that can be arranged in boxes of 6, 9, 12, 10, 15, 20, 21, 24, 27, 30, 40, or 50 each, so that there may be none remaining; also find the L.C.M. of 7, 14, 21, 28, 35, 42, 49.

iii. Reduce $\frac{3913}{391391}$ to its simplest form; and express the fractions $\frac{40}{317}$, $\frac{73}{1846}$ with the denominators 352187 and 27690 respectively.

(2) i. Express in a fractional and decimal form the result of $3\frac{1}{3}$ of $1\frac{2}{3}+2\frac{3}{4}-1\frac{1}{4}$ of $5\frac{1}{2}-\frac{11}{18}-1\frac{4}{9}$ of $\frac{1}{3}$.

ii. The sum of two numbers is $19\frac{17}{84}$ and the greater is equal to the difference between the sum of $3\frac{1}{2}$, $\frac{14}{15}$, and 11 and the sum of $2\frac{1}{7}$, $\frac{13}{14}$, $\frac{2}{3}$, and $\frac{29}{30}$; find the less number, and express it as a decimal.

iii. Simplify ($2\frac{5}{14}$ ton $+\frac{7}{8}$ of 2 cwt. $+\frac{13}{16}$ of a qr. $+14\frac{3}{8}$ lbs. $+\frac{1}{7}$ ton) $-$ ($3\frac{5}{7}$ cwt. × $6\frac{5}{8}$).

(3) i. A gentleman distributed tracts to the value of 33·264*l.*, and the average cost of each tract was ·0875*s.*; how many were there?

ii. What decimal multiplied by the product of 27·34 and 13·037 will give 3·2007555884?

iii. The quotient obtained by dividing a certain number by ·017 is $376470{\cdot}\dot{5}8823529411647\dot{0}$; find the dividend.

(4) i. Express as equivalent vulgar fractions $17{\cdot}060\dot{7}1428\dot{5}$; $3{\cdot}\dot{5}3846\dot{1}$; $10{\cdot}013541\dot{6}$.

ii. What vulgar fraction is equivalent to (219·283 + 71·4 + 73·68 + 180·9) − (16·2598 + ·3037 + 201·5 + 27 + 300·1604375)?

iii. What is the value of 5·075*l.* + (3*l.* 13*s.* 4*d.* × ·08175) + 27·625 of 50*l.*? and subtract the sum from the product of 73*l.* 12*s.* $3\frac{5}{8}$*d.* by $39\frac{3}{8}$.

(5) i. A mercantile firm failed for 97361*l.* 12*s.* 6*d.*, and their effects realised sufficient to declare a dividend of 13*s.* $7\frac{1}{2}$*d.* in the £; find the amount of their assets

ii. Find the rent of a farm of 208 acres 3 roods 27 poles, at 16*s.* 8*d.* per acre; also find the farmer's net income, which is equal to four-fifths of the rent.

iii. Bought 35 tons 10 cwt. 2 qrs. 19 lbs. of lead at 2*l.* 14*s.* per cwt., and gave in part payment 418 cwt. 3 qrs. 27 lbs. of iron at 1*l.* 10*s.* 4*d.* per cwt.; find the balance.

(6) i. Find the net weight of 104 barrels, weighing gross 726 cwt. 18 lbs., allowing $16\frac{1}{2}$ lbs. for the weight of each barrel.

ii. What is the net weight of 24 hhds. weighing gross 624 cwt., tare 28 lbs. per cwt., tret 4 lbs. in 104 lbs., and draft 2 lbs. in 3 cwt.?

iii. Find the net value of 200 firkins of lard, weighing gross 245 cwt. 1 qr. 14 lbs., tare 16 lbs. per cwt., and tret 4 lbs. in 104 lbs., at $6\frac{3}{4}$*d.* per lb.

(7) i. What sum will 917*l.* 1*s.* 3*d.* amount to at Simple Interest, in 5 yrs. 125 days, at $3\frac{1}{3}$ per cent.?

ii. What sum will 333*l.* 12*s.* 6*d.* produce as interest in 9 yrs. 12 wks., at $8\frac{1}{8}$ per cent.?

iii. What sum lent at Compound Interest for 3 yrs., at $7\frac{1}{2}$ per cent., will amount to 439*l*. 9*s*. $3\frac{3}{840}$*d*.?

(8) i. A bought goods from B and gave him in part payment a Bill of 32*l*. 5*s*. $10\frac{3}{4}$*d*., which had 5 months to run; what sum should B allow to A for the said Bill, the rate of discount being $7\frac{1}{5}$ per cent.?

ii. Find the discount and present value of a Bill of 292*l*. 13*s*. $0\frac{3}{4}$*d*. due in 11 months, at $3\frac{1}{2}$ per cent.

iii. A Bill of 615*l*. 18*s*. 9*d*. drawn on April 6, at 6 months, is discounted on June 4th; find the bankers' discount at $6\frac{1}{2}$ per cent. per annum.

(9) i. A buys 650*l*. Stock at $76\frac{1}{2}$, and sells it at $95\frac{5}{8}$; how much does he gain, and what per cent.?

ii. Find the value of a legacy of 10,000*l*. in the Great Western Stock at $84\frac{1}{2}$, the duty being 3 per cent., and the expenses $1\frac{1}{2}$ per cent.

iii. At what price can I transfer Stock from the 4 per Cents. to the 5 per Cents. at 120, in order to gain 5 per cent. of income?

(10) i. Sold sugar at 34*s*. per cwt., and lost 15 per cent.; what should I have gained or lost per cent. by selling it at 42*s*.?

ii. I have sugar, for which I gave 40*s*. per cwt. and which I wish to sell for 30*l*., to cover the expenses of warehousing and duty, at $\frac{5}{8}$*d*. per lb., and to clear 5 per cent.; how much have I?

iii. A grocer sold a mixture of sugar, containing three sorts; the price of the first was 5 per cent. less than that of the second, and that of the second 4 per cent. less than that of the third; in what proportion did he mix them, so as neither to gain nor lose?

(11) i. A and B wish to settle a debt of 200*l*. owing to C. A has cloth at 15*s*. per yard, and B has linen at 2*s*. 4*d*. per yard. A's portion of the debt is 30 per cent. more than B's; what goods do they give?

ii. A has 1500 lbs. of coffee, at 1*s*. 4*d*. per lb., which he barters with B for sugar at 6*d*. per lb. and rice at 5*d*.

per lb. He is to have $\frac{1}{3}$ the value of his coffee in money; how many lbs. of sugar and rice does he get, and of each an equal quantity, and how much money?

iii. A bought 350 quarters of wheat at 28*s.* per quarter, and kept it 4 months, during which it lost 4 per cent. of its measure. He exchanged it for 2050 doz. of ale, at 5*s.* 6*d.* per doz.; what did he gain, reckoning interest at 5 per cent.?

(12) i. A rate of 4*l.* 13*s.* 9*d.* per cent. is levied on three parishes, the estimated value of each being respectively 6400*l.*, 7200*l.*, and 2200*l.*; find what portion of the rate is paid by each, and what a person will pay who holds an estate worth 572 guineas.

ii. Two bankrupts, whose assets are respectively 750*l.* and 6975*l.* 7*s.* 9*d.*, compound with their creditors, and pay severally 12*s.* 6*d.* and 15*a.* 6*d.* in the £1; what were their debts?

iii. Two parcels of rice, precisely similar, are sent to A, B, and C. A is to have $\frac{3}{4}$ of one parcel, B $\frac{2}{3}$ of the other, and C the remainder of the two. A, however, purchases $\frac{4}{7}$ of C's share, and B $\frac{2}{3}$ of the remainder; show how to divide the parcels that only one may be opened.

(13) i. 100*l.* is due in 2 months 3 wks. 4 days, 200*l.* in $9\frac{1}{2}$ wks., and 300*l.* in $4\frac{1}{2}$ months; find the equated time.

ii. A owes 2500*l.* to be paid in half-yearly instalments of 250*l.* each, bearing interest at $3\frac{1}{2}$ per cent. per annum; find the tenth payment, and the equated time.

iii. If in (ii.) A can make 5 per cent. of his money, what would he lose by paying the debt in the equated time.

(14) i. What decimal multiplied by the square root of 567148·01231929 will produce 4·44324457?

ii. Find the product of the square root of 257362·421481, and the cube root of ·000105958111168.

iii. Find the number of acres in a field equivalent to the area of the surface of a cube whose content is 45410136 c. yds. 26·927 c. ft.

15) i. Exchange 43856·412 francs for sovereigns, at 25·90 francs per 1*l.*

ii. The exchange between London and Paris is 25·30 francs per 1*l.*, and between Paris and Hamburg 1 mark per 1·8 francs; what is the exchange between London and Hamburg?

iii. A in London has a Bill of 200*l.* on B in New York, and C in Paris has a Bill on A to the amount of 20,000 francs, and B on C to the amount of 1500 dollars. The exchanges are 1*l.* = 25·30 francs; 4*s.* 6*d.* = 1 dollar; and 6·15 francs = 1 dollar; find the differences to be paid to A and C by cancelling these Bills, if B settles with C, and C with A.

PAPER XXI.

(1) i. Simplify $\dfrac{5\frac{1}{2} \text{ of } 4\frac{3}{4}}{2\frac{3}{5}} \times \dfrac{\frac{33}{44}}{1\frac{1}{17}}$ of $\frac{72}{99}$ of $10\frac{1}{2}$ of £$\frac{1}{17}$.

ii. My share of a vessel is $\frac{27}{44}$ of $\dfrac{3\frac{3}{5} \text{ of } \frac{11}{21} \text{ of } 2\frac{4}{9}}{3\frac{3}{7} \text{ of } 1\frac{1}{2} \text{ of } \frac{8}{35}}$ of $\frac{1}{11}$ of the whole, and for this I paid 4363*l.* 12*s.* 6*d.*; find the value of the vessel.

iii. A, B, C, D entered into partnership; A furnished $\frac{2}{9}$ of the capital, B $\frac{3}{10}$ of the remainder, C $1\frac{1}{3}$ of B's, and D 15404*l.* 0*s.* 6*d.*; what fraction of the whole did each supply, and what was its value?

(2) i. Add together $5\frac{5}{8}$, $\frac{3}{10}$, $\frac{11}{16}$, $7\frac{1}{15}$, and from the sum subtract $3\frac{3}{4} + \frac{11}{30} + \frac{13}{24} + 3\frac{5}{9} + \frac{1}{2}$.

ii. Express in the simplest form $\dfrac{4\frac{3}{4} + \frac{7}{12}}{8\frac{1}{6} - 3\frac{7}{10} + \frac{5}{8}}$ of $\dfrac{\frac{\frac{7}{8} \text{ of } 3}{1\frac{4}{5}}}{1\frac{5}{6} + 3\frac{11}{18}}$.

iii. After paying $7\frac{13}{480}$*l.*, $\frac{11}{320}$ of 4*l.*, $\frac{5}{8}$ of a shilling, $\frac{2}{3}$ of a crown, and $1\frac{3}{8}$ of 5 half-crowns, what remained out of $4\frac{5}{9}$ of $3\frac{7}{8}$*l.*?

(3) i. What decimal is equivalent to the sum of $10{\cdot}\dot{7}1428\dot{5}$, $5\,909\dot{0}$, $1{\cdot}22\dot{7}$ and $2{\cdot}\dot{6}$?

ii. Divide 1*l.* 17*s.* 4½*d.* by 3·7875; multiply 5 guineas by $\cdot 389\dot{2}8571\dot{4}$, and add the two results.

iii. Multiply 2 tons by 33·0125; find in positive terms ·00375 of a cwt.; divide 2 qrs. by $\frac{1}{27\frac{3}{4}}$, and give the sum of the three results, which subtract from 100 tons.

(4) i. How many lbs. are there in $\cdot 7389\dot{2}8571\dot{4}$ of ½ a ton?

ii. Simplify

$$\left(\frac{1l.\ 13s.\ 7\tfrac{1}{2}d.}{\cdot 33625}+\frac{2l.\ 0s.\ 10\tfrac{1}{2}d.}{\cdot 389\dot{2}8571\dot{4}}+\frac{17s.\ 10\tfrac{3}{4}d.}{\cdot 894791\dot{6}}\right)-\left(\frac{5s.\ 9\tfrac{3}{4}d.}{\cdot 58125}\times\frac{\cdot 006846}{\cdot 00125}\times\frac{724\tfrac{3}{8}}{312\tfrac{1}{2}}\right)$$

iii. A bachelor bequeathed his property in the following manner:—$\cdot 02\dot{7}$ of it to each of 9 nephews, $\cdot 01\dot{6}$ of it to each of 8 nieces, $\cdot 0\dot{2}$ of the remainder to each of 15 cousins, and the rest, which amounted to 12284*l.*, among his four brothers, in proportion to their ages, which were respectively 53, 57, 63, 67; what was the total amount bequeathed, and also the sum left to each person?

(5) i. A bought 9137 cwt. of currants at 3*l.* 17*s.* 9½*d.* per cwt. from B, and B bought from A 5084 firkins of butter at 6*l.* 16*s.* 7¾*d.* per firkin; how much is the balance, and to whom due?

ii. Find the value of a ship's cargo consisting of 1475 cwt. 1 qr. 14 lbs. of iron ore at 5*l.* 14*s.* 6*d.* per ton, and 645 cwt. 2 qrs. 26 lbs. of lead ore at 1*l.* 15*s.* per cwt.

iii. A ship foundered at sea with 1347 bales of cotton averaging 5 cwt. 2 qrs. 12 lbs., and 729 bales of silk weighing 13 tons 2 qrs. 25 lbs. The ship was valued at half the cargo, the cotton was worth 1*s.* 6¾*d.* per lb., and the silk 99*l.* 3*s.* 4*d.* per cwt.; find the value of each, and the total loss.

(6) i. Find the net weight of 77 hhds., each 37 cwt. 1 qr. 7 lbs. gross, tare 14 lbs. per hhd., and tret 4 lbs. in 104 lbs.

ii. What sum must I remit to a sugar planter in Jamaica for a shipment of 73 hhds. each 16 cwt. 24 lbs. gross, at 3¼*d.* per lb., 1 qr. 17 lbs. being allowed for the weight of each hhd.?

iii. How many lbs. net in 13 bags of coffee, weighing 496 cwt. 1 lb., allowing 12 lbs. per cwt. for the weight of the bags, 2 lbs. in 100 lbs. for loss in roasting, and 2 lbs. in 210 lbs. for loss in grinding, &c.?

(7) i. I lent 180*l.* 19*s.* 7*d.* out at interest at 6½ per cent. per annum, and it was repaid to me at the expiration of 5 yrs. 125 days; what did I receive annually as interest, and how much altogether?

ii. In what time will the interest of 840*l.* 13*s.* 4*d.* become 83*l.* 12*s.*, $1\frac{163}{365}$*d.*, at $3\frac{23}{73}$ per cent.?

iii. Find the Compound Interest of 3333*l.* 6*s.* 8*d.* for 2½ yrs. at 6 per cent., interest payable half-yearly.

(8) i. If on the 1st of June you took to a bank to be discounted a bill of 695*l.* 18*s.* 9*d.* which was not legally due until the 6th of January, how much would the banker give you, the rate of discount being 3⅓ per cent.?

ii. How much does a banker deduct more than is really his due on a bill of 787*l.* 10*s.*, which has 7 months to run, at 10 per cent.?

iii. What sum is the present value of 1138*l.* 5*s.* 1¾*d.* due in 292 days, at 3¾ per cent.?

(9) i. If I buy 2000*l.* Stock for 1800*l.*, and get an income of 30*l.* a year, what is the interest per cent. on my capital, and the price of Stock?

ii. I buy Stock which pays 4½ per cent. per annum, half-yearly; but for the second dividend it paid 4¾ per cent. If I bought it at first for 84, what ought I to get for it at the end of 12 months? Supposing I had bought 1000*l.* of it, what would my money amount to with the interest?

iii. 10000*l.* was invested in 4½ per Cent. Stock at 99. It was afterwards transferred to the 3 per Cents. at 72, and then to the 5 per Cents. at 105; what was the change in income at each transfer?

(10) i. If I get 2*l.* for an article, what did I give for it if I sell it for 1¾ of the cost?

ii. If I get 2*l.* 4*s.* for an article, and gain $\frac{1}{10}$ of the cost, at what price should I have sold it to gain 3½ per cent.?

iii. Mixed three kinds of sugar in the proportion of 26¼ lbs., 25 lbs., and 24 lbs. respectively. The price of the mixture was 7*d.* per lb., which was a profit of 5 per cent. on the cost of the third sort; what were the prices of the sugar, supposing the second 4 per cent. lower than the third?

(11) i. Bartered tea at 6*s.* per lb. for coffee at 1*s.* 6*d.* per lb., the ready-money price of which was 1*s.* 3¾*d.*; what was the ready-money price of the tea?

ii. Bartered coffee at 1*d.* per lb. profit for tea which was given in exchange at 5*s.* 6*d.* per lb., being a profit of 5 per cent. on the ready-money price; what did the coffee cost per lb.?

iii. Bartered 2560 yds. of cloth, at 2*s.* 6*d.* per yard, being 2*d.* per yard profit on the ready-money price, for 1666$\frac{42}{43}$ lbs. of sugar which cost 4*d.* per lb., together with a quantity of rice and tea. The weight of the tea was ⅛ that of the rice, and the weight of the rice half as much again as that of the sugar. The cost price of the rice was 2*d.* per lb.; how much tea was there, and what was the cost price of it per lb.?

(12) i. A and B speculate and gain 100*l.*, which is 6½ per cent. profit on their capital. A's share is 20*l.*; what was the capital of each?

ii. A and B each begin trade with 200*l.*, and gain 1¼ per cent. per month. After 4 months C enters; what must be his capital, that he may share equally in the gain at the end of 12 months?

iii. A and B each begin with 500*l.*, and they gain 12½ per cent. per annum. In the fourth year they lost 200*l.* each; what were they each worth in 7 years?

(13) i. 1000*l.* is paid as follows: 250*l.* in 3 months, 325*l.* 10*s.* 6*d.*

in 5 months, 184*l*. 13*s*. 4*d*. in 6 months, and remainder in 10 months; find the equated time.

ii. A borrowed three sums of 40*l*., 200*l*., and 400*l*. for 2 months, 5 months, and 8 months respectively. For the first two months he gained 2 per cent. by speculation, but after that he gained as much per cent. per annum as to pay off all the debts in 8 months, and gain the first one; what was the per centage of his gain after the second month, supposing he paid 5 per cent. interest for the Bills when overdue?

iii. A debt of 1000*l*. is to be paid by monthly instalments of 20*l*. The first five were paid regularly, and then the instalments were omitted for 12 months; what sum would then commute the arrears and interest, as well as the remainder of the debt, reckoning interest at 5 per cent. per annum?

(14) i. What number, when multiplied by itself, will give 54761376408649? Find also the square of ·006357904.

ii. The cube of a certain number is 300886·913413418688; find the number.

iii. Find the cube of ·005537 and the cube root of 300·1 to four figures.

(15) i. Exchange 975*l*. sterling into Grecian drachmai; 1*l*.= 28·30 drachmai.

ii. A, in London, owes B, in Paris, 5060 francs, for which he sends 200*l*. in English notes, and these B finds are at a discount of 2½ per cent. B owes C, in London, 150*l*., for which he gives him a French Bill, and C finds French money to be at a discount of ·12 francs from the par of Exchange, which is 25·30 francs; how much will A have to pay B, and B to pay C additional?

iii. A, in Holland, has Bills on B, in London, to the amount of 4500 florins. C, in Paris, has Bills on A to the amount of 12500 francs; and B has Bills on C of 560*l*. They cancel Bills as far as possible, paying differences; how do they settle, if 25.60 francs = 1*l*., and 57.25 florins = 120 francs?

PAPER XXII.

(1) i. What fraction of 2*l.* 10*s.* is 15*s.* $10\frac{1}{2}$*d.*? Express 3 cwt. 3 qrs. 16 lbs. in terms of $1\frac{1}{2}$ tons.

ii. From $15\frac{3}{5}+14\frac{1}{7}+\frac{10}{21}$ take the sum of $\frac{11}{14}$, $3\frac{1}{35}$, $13\frac{17}{42}$, and $\frac{5}{8}$, and divide the remainder by the continued product cf $4\frac{4}{5}$, $\frac{7}{10}$, $\frac{13}{14}$ of 60, $\frac{11}{26}$, $13\frac{1}{2}$, and $\frac{1}{9}$ of $\frac{15}{22}$.

iii. From the sum of $\frac{3}{11}$ of 17*s.* 5*d.*, $\frac{9}{13}$ of 1*l.* 1*s.* $11\frac{1}{4}$*d.*, and 9*s.* 8*d.* × $\frac{3}{16}$, take $\frac{5}{19}$ of 1*l.* 4*s.* $11\frac{1}{4}$*d.*, and express the result as a fraction of 5*l.* 10*s.* $1\frac{5}{16}$*d.*

(2) i. What number added to $\dfrac{2\frac{3}{5}\text{ of }7\frac{3}{91}}{\frac{8}{21}\text{ of }112\text{ of }\frac{39}{119}}$ will produce the difference between $2\frac{10}{13}+12\frac{1}{65}+\frac{1}{8}+1\frac{1}{15}$, and the sum of $\frac{13}{20}$, $5\frac{17}{30}$, and 2?

ii. Simplify $\left(\dfrac{1{\cdot}\dot{3}\times3{\cdot}75\times{\cdot}5}{2\frac{4}{7}-({\cdot}\dot{6}\times{\cdot}\dot{8}5714\dot{2})}-\dfrac{\frac{3{\cdot}\dot{3}}{1{\cdot}0\dot{6}}}{1\frac{1}{9}+3{\cdot}\dot{5}7142\dot{8}+\frac{20}{63}}\right)+$

$\dfrac{3\frac{3}{8}\text{ of }3\frac{3}{4}\text{ of }{\cdot}7\dot{2}\text{ of }{\cdot}\dot{3}9506172\dot{8}}{3\frac{4}{7}\text{ of }1{\cdot}1\dot{3}\text{ of }1\frac{2}{5}\text{ of }\frac{1}{17}}$, giving the result in a fractional and decimal form.

iii. A tradesman lost $\cdot8\dot{3}$ of $\cdot9$ of $\frac{3}{4}$ of his capital in the first year that he was in business, in the second year he lost $\cdot\dot{8}$ of $\cdot6\dot{4}2857\dot{1}$ of $\cdot75$ of $\frac{2}{3}$ of what remained, in the third year he gained $\cdot875$ of $\cdot\dot{9}$ of $\frac{11}{14}$ of $\frac{8}{9}$ of his original capital, in the fourth year he gained $\cdot\dot{3}8461\dot{5}$ of the capital which he had at the commencement of that year, and then the difference between his capital and that with which he began was 726*l.*; was he a gainer or loser, and what was his original capital? Also find the amount of his losses or gains in each year.

(3) i. The product of two numbers is 7·609152672, and one of the factors is 219·4104; find the other.

ii. The sum of two numbers is 1277·757, and their difference is 222·243; find the numbers, and divide ·598234 by the greater.

iii. Divide 43*l.* 6*s.* 0·525*d.* by 7·8375, to the quotient add $2\frac{31}{40}$*l.* × 15·625, and from this sum subtract the value of

3·775 yds. at 1*l.* per yd., ·00625 of a ton at 1 guinea per ton, and 3·0875 ozs. of gold at 5*l.* per oz.

(4) i. What decimal of a league are 3 furlongs 10 poles 3 yds.?

ii. Bought $\dfrac{\frac{7}{11} \text{ of } 2\cdot 1\dot{6} \text{ of } \frac{3}{19}}{\cdot\dot{6}3 \text{ of } \frac{75}{76} \text{ of } 1\cdot 7\dot{3}}$ of a business, and sold $\dfrac{1\cdot\dot{8}}{\frac{1}{6}}$ of $2\frac{14}{15}$ of $2\frac{4}{7}$ of $\frac{3}{22}$ of $\frac{5}{68}$ of my share for 8238*l.*; find the value of my original share, and also of the whole business.

iii. Express in positive terms $\frac{13}{28}$ of a cwt. + (5 tons × 2·875) + ·34375 of a qr. + (28 lbs. × 2·625) + ($1\frac{1}{8}$ cwt. × 13·53125).

(5) i. Find the value of $3523\frac{11}{16}$ bales of cotton, at 20*l.* 10*s.* $10\frac{1}{4}$*d.* per bale.

ii. Two ingots of gold weighed respectively 7 lbs. 8 oz. 18 dwts. 7 grs. and 14 lbs. 7 oz. 16 dwts 21 grs.; what did they cost, the former at 3*l.* 10*s.* per oz., and the latter at 3*l.* 17*s.* $10\frac{1}{2}$*d.* per oz.?

iii. What must be given for 15 cwt. 2 qrs. 24 lbs. of silver ore, at 128*l.* 6*s.* 8*d.* per ton? Also for 24 gold cups weighing each 9 oz. 10 dwts. 16 grs., at 54*l.* 10*s.* per lb.?

(6) i. How many cwt. net in 16 hhds., each 15 cwt. 3 qrs. 8 lbs., allowing 14 lbs. per cwt. for the weight of the hhds., and tret 4 lbs. in 104 lbs.?

ii. Find the net value of 93 hhds. of palm oil, each 17 cwt. 1 qr. 17 lbs., at 7*s.* $3\frac{1}{4}$*d.* per stone of 14 lbs., allowing 40 cwt. 3 qrs. 27 lbs. for the weight of the hhds.

iii. What is the net weight of 56 hhds., each 12 cwt. 3 qrs. 18 lbs., allowing $17\frac{1}{2}$ lbs. per cwt. for tare, 4 lbs. in 104 lbs. for tret, and 2 lbs. in 252 lbs. for draft?

(7) i. At what rate per cent. will 315*l.* 18*s.* $6\frac{1}{2}$*d.* become 410*l.* 14*s.* $1\frac{1}{4}$*d.* in 5 yrs.? How much is one year's interest?

ii. On the 3rd of August 1863, I lent 827*l.* 6*s.* 8*d.* out at interest at $3\frac{3}{4}$ per cent., and the loan was repaid on the 5th of March 1864; how much did I receive for principal and interest?

iii. What sum will 159*l.* 7*s.* 6*d.* amount to at Compound Interest for 2¾ yrs., at 4½ per cent.?

(8) i. Find the present worth of a Bill of 569*l.* 15*s.* 6¾*d.* due in 2 yrs. 3 months, at 3 per cent.

ii. Find the discount on a Bill of 1022*l.* 11*s.* 0¼*d.* due 95 days hence, at 6¼ per cent.

iii. What will a banker gain by discounting a Bill of 475*l.* 4*s.* which has 7½ months to run, at 5 per cent. per annum?

(9) i. The sum of 9040*l.* 16*s.* is placed in the 3½ per Cents. at 94; find the income obtained, allowing on the Stock purchased ⅛ per cent. to the broker, and $\frac{1}{20}$ per cent. for other expenses.

ii. A person invests 3000*l.* in the 3 per Cents. at 94⅞, and 2000*l.* in 6 per Cent. Canada Bonds at 112⅛, allowing in each case ⅛ per cent. on the purchase to the broker; find the average per centage obtained on the 5000*l.* invested.

iii. Find the sum which must be invested in the 3 per Cents. at 90, at Simple Interest, payable half-yearly, to amount in 23½ yrs. to 2317*l.*, the price of the funds remaining unchanged.

(10) i. A grocer has equal quantities of two kinds of tea worth 5*s.* and 4*s.* per lb. respectively; he takes ⅓ of each kind, and mixes it with the remainder of the other; at what price per lb. ought the two mixtures to be respectively sold?

ii. Two persons buy apples at a penny each; one sells them at 5 for 6*d.*, and the other at 6 for 7*d.*; compare their gains per cent.

iii. A merchant sells tea to a tradesman at a profit of 60 per cent., but the tradesman becoming bankrupt pays only 2*s.* 6*d.* in the pound; how much per cent. does the merchant gain or lose by his sale?

(11) i. Supposing each gallon contains 277 cubic inches, and that a cubic foot of water weighs 1000 ozs., show that the popular rule,

A pint of water
Weighs a pound and a quarter,

is nearly true.

ii. If a person invest 1210*l.* in the $3\frac{1}{4}$ per Cents. at $97\frac{1}{2}$, and a like sum in the 5 per Cents. at $90\frac{3}{4}$, what will be his income?

iii. Reduce to its simplest form $\dfrac{\frac{1}{2}\{\frac{1}{3}-\frac{1}{14}+\frac{6}{7}(\frac{1}{2}-\frac{1}{3})\}}{1+\dfrac{1}{2+\frac{1}{3}}}$.

(12) i. A began business with 150*l.*; after a time B joins him with 450*l.* They share equally at the end of 12 months; when did B join?

ii. A puts into a business 200*l.*, and B 300*l.*; how much should C put in at the end of 4 months, that his share of the gain may equal A's and B's together at the end of 12 months?

iii. How should A, B, C share 90*l.* if their capitals were the same, but the time of investment 2 months, 3 months, 4 months respectively?

(13) i. If 25 in the scale whose radix is 10 be equivalent to 27 in another scale, find the radix of the latter.

ii. If 12 farthings = 1*d.*, and 12*d.* = 1*s.*, and 12*s.* = 1*l.*, then multiply 23*l.* 3*s.* 9*d.* by 3*l.* 5*s.*

iii. If a foot = 10 inches, and an inch = 10 parts, and so on, find the side of a square board which contains 5 feet 4 inches $7'.5''.6'''$.

(14) i. The square root of a certain number is 6430729; find the number. Also extract the square root of 1285291·561849.

ii. Find the square root of 6735·937241835947 to six places of decimals, and prove it.

iii. Find the area of a square field whose side is of the same length as that of a cube containing 525636096·753664 cubic yards.

(15) i. Gained $12\frac{5}{8}$ per cent. by selling apples at 10*d.* per dozen; how much per cent. is gained by selling them at 1*s.* 5*d.* per score?

ii. If A and B together can do $\frac{4}{5}$ of a piece of work in $9\frac{1}{2}$ days, and A and C could together do $\frac{5}{8}$ of it in 12 days; in what time could they do it altogether, if B's power of work were equal to A's, and he worked twice as long as the others?

iii. Bought 684 yds. at 14*s.* 9*d.* per yard; sold $\frac{1}{3}$ of them at 15*s.* 7*d.* per yard, $\frac{1}{4}$ at 17*s.* 2*d.*, and the remainder at 14*s.* 1*d.*; find the gain per cent.

PAPER XXIII.

(1) i. Find the G.C.M. of 1292, 2516 and 697; the highest common divisor of 21505, 302005 and 2057; also the greatest number common to 13575 and 2353.

ii. Find the least number of slate pencils that could be made up exactly in packages of 7, 10, 12, 24, 14, 18, 20, 30, or 35; also find the L.C.M. of 7, 10, 11, 13, 15, 18, 22, 26, 30.

iii. Reduce $\frac{7346984375}{13215}$ to a mixed number; $1948\frac{339}{5968}$ to an improper fraction; and express $\frac{15947}{42669}$ in its simplest form.

(2) i. Find the continued product of $\left.\begin{matrix} \\ \end{matrix}\right\} \dfrac{4\frac{3}{13}}{1\frac{4}{17}}$ of $\dfrac{13 \times 18 \times 7}{110 \times 17 \times 9}$ of $\dfrac{3\frac{1}{5} \text{ of } \frac{27}{56}}{\frac{1}{15}}$ of $5\frac{5}{13}$, and multiply the result by the difference between $\frac{5}{28}$ and $\frac{54}{175}$.

ii. Bought a mansion and an estate for 120,000*l.*, of which the mansion is worth $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{1}{2}$, and the wood on the estate $\frac{9}{10}$ of $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{1}{3}$; find the value of the mansion, the wood, and the land.

iii. In a cricket match one side of 11 obtained a certain number of runs; the highest scorer made $\frac{5}{6}$ of $\frac{9}{10}$ of $\frac{2}{3}$ of $\frac{5}{8}$ of the whole, the next highest ran up a score of $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{2}{5}$ of $1\frac{1}{3}$ of the whole; each of two others contributed $\frac{7}{9}$ of $\frac{3}{4}$ of $\frac{1}{7}$ of the whole, each of three others obtained $\frac{7}{8}$ of $\frac{2}{5}$ of $\frac{1}{7}$ of the number, the remaining four retired with a 0 to their names, and there were 25 byes and wides; find the total score.

(3) i. Express 1 ton 16 lbs. in terms of 2 cwt. 2 qrs.; and 5 fur. 12 poles 3 yds. in terms of a league, giving both in a fractional and decimal form.

ii. Express in a decimal form the sum of 3·075, $11{\cdot}16\dot{6}$, $\frac{8}{15}$ and $9{\cdot}\dot{4}28571$.

iii. Express in positive terms ·121875 of a ton $-\frac{39}{32}$ of a cwt. $+ 5\frac{11}{160}$ of a ton $-$ ·06640625 of a quarter $- 5\frac{5}{8}$ of a lb. $-$ ·1875 of an oz.

(4) i. Simplify 956·34 + 7 thousandths − 956 tenths − 347 ten-thousandths − 9·563 − 47 hundred-thousandths − ·9503 − 6047 millionths + 76234 thousandths + 7623·4 − 585 − 585 thousandths + 585 tenths − 7234 hundred-thousandths − 36·7 + 784·487 − 3727 hundredths − 94 ten-thousandths.

ii. A ship worth 15,375*l.* is sold for $\frac{11}{16}$ of her value; how much did a shareholder realise who owned ·625 of $\cdot 8\dot{6}$ of $\cdot\dot{8}0769\dot{2}3$ of $\dfrac{\frac{16}{17}}{1\frac{1}{34}}$ of $\dfrac{\cdot 875}{1\frac{1}{19} \text{ of } 2\frac{17}{20}}$ of her?

iii. If an iron-clad vessel of war cost 80,000*l.* building, $\frac{2}{5}$ as much equipping, and $\frac{1}{8}$ as much storing; find the cost of 12 such vessels, 5 per cent. being saved on the above cost in the building, 17½ per cent. in the fitting, and 15 per cent. in the storing.

(5) i. Find the value of 2099 bales of woollen goods, each containing 35 pieces at 11*l.* 13*s.* 9¼*d.* per piece.

ii. What must be given for 32 tons 15 cwt. 1 qr. $21\frac{7}{11}$ lbs. of silver ore, at 2*l.* 13*s.* 11¼*d.* per lb.?

iii. A sold to B 54 cwt. 3 qrs. 24 lbs. of sugar at 2*l.* 12*s.* 6*d.* per cwt., and owed him for the rent of 95 acres 2 roods 18 poles of land, at 16*s.* 8*d.* per acre; how much is the balance, and to whom due?

(6) i. Bought 100 bags of coffee weighing gross 193 cwt. 1 qr. 14 lbs.; what did they cost me at 1*s.* 4½*d.* per lb., allowing 16 lbs. per cwt. for the weight of the bags, and 4 lbs. in 104 lbs. for tret?

ii. How many 4 lb. tins of mustard may be filled out of 16 hhds. each 14 cwt. 3 qrs. 12 lbs. gross, allowing

$17\frac{1}{2}$ lbs. per cwt. for the weight of the hhds., and 4 lbs. in 104 lbs. for tret?

iii. Find the net weight of 37 hhds. weighing gross 556 cwt. 2 qrs. 9 lbs., tare 1 cwt. 1 qr. 11 lbs. per hhd., tret 3 lbs. in 100 lbs., and draft 2 lbs. in 338 lbs.

(7) i. I borrowed a sum of money at $4\frac{3}{8}$ per cent. Simple Interest, and in 6 yrs. 3 months I repaid the same with interest amounting to 409*l.* 1*s.* $10\frac{1}{2}$*d.*; how much was the loan?

ii. For the loan of a certain sum for 11 yrs. 8 months, at 5 per cent., I paid as Simple Interest 562*l.* 14*s.* $8\frac{1}{4}$*d.*; how much did I borrow?

iii. In what time will the interest of 656*l.* 5*s.* become 110*l.* 14*s.* $10\frac{1}{8}$*d.* at 3 per cent. per annum?

(8) i. What is the present value of 89*l.* 9*s.* $9\frac{1}{2}$*d.* due in $5\frac{1}{11}$ yrs. at $8\frac{1}{4}$ per cent. per annum?

ii. Find the bankers' discount on a Bill of 1300*l.* 6*s.* 3*d.* due in 175 days, at $3\frac{3}{4}$ per cent.

iii. A Bill of 8076*l.* 2*s.* 11*d.* is due as follows: viz., $\frac{1}{4}$ in 4 months, $\frac{1}{5}$ in 6 months, $\frac{3}{10}$ in 8 months, and the rest in 10 months; find the present worth of the whole, reckoning 5 per cent. discount.

(9) i. A man bought 500*l.* 3 per cent. Consols, when they were at $93\frac{1}{2}$, and after receiving one dividend, he sold at $96\frac{3}{4}$; what did he gain by the transaction?

ii. If money invested in the 3 per cent. Consols give exactly 3 per cent. after the payment of 1*s.* in the pound income-tax, find the price of the Consols, allowing $\frac{1}{8}$ per cent. to a broker for the purchase.

iii. The price of 3 per cent. Consols is $90\frac{3}{8}$, what sum must be invested in order to have an income of 24*l.* per annum? and what is the rate of interest on the money invested?

(10) i. A grocer buys some tea at 4*s.* per lb., and some at 5*s.* 6*d.* In what proportion must he mix them, that when he sells the tea at 6*s.* per lb. he may gain 20 per cent.?

ii. A bookseller sells 3000 copies of a book at 41*s.* and takes in exchange a Bill at 6 months for the amount; how much would he gain or lose by selling them instead, for cash at 39*s.*, interest being at 5 per cent. per annum?

iii. A wine merchant buys two sorts of spirit at the rate of 16*s.* 6*d.* and 10*s.* 6*d.* per gallon respectively. He mixes them in the proportion of 5 parts of the cheaper to 7 of the dearer; at what rate per gallon must he sell the mixture to make 25 per cent. profit?

(11) i. If the wages of 54 men for 36 days amount to 60*l.* 15*s.*, what will be the wages of 30 men and 50 boys for 32 days, supposing 2 men to do as much work as 5 boys?

ii. How many hours a day must 171 men work to do in 91 days what it takes 143 men 133 days of nine hours each to do?

iii. Workmen can perform a certain labour in a week, if they work 11 hours a day for 6 days; how many hours a day must they work to perform the same in the same time, if they take half of Saturday as a holiday, but do a twelfth more work each hour?

(12) i. A Banking Company fails for 50,000*l.*; the assets are 7500*l.* If there are 20 shareholders with each an equal stake, what must each contribute to make up the difference?

ii. If in (i.) five of the shareholders were bankrupt, and paid each 7*s.* 6*d.* in the pound; how much would each of the remaining fifteen have to pay to make up the deficiency?

iii. A, B, and C, make up a capital of 1800*l.* in sums proportionate to the numbers 2, 3, 4. In one branch of their trade they lose 2½ per cent. of their capital, in another they gain 10 per cent., and in the third they gain half as much as they gain altogether on the other two; how much did they gain and how did they share it?

(13) i. Find the cube root of 32242 in the scale whose radix is 5.

ii. A vessel containing 630 gallons is filled by 3 spouts, A, B, C. A fills $\frac{2}{3}$ of it in $1\frac{1}{2}$ hour, B and C together fill it in 30 minutes, and B pours in $\frac{1}{10}$ as much again as C; how many gallons does each discharge in 1 minute?

iii. If, in a battle, one rifle in fifteen kills a man every time a volley is fired, and a rifled cannon does the work of 10 rifles and bears the same proportion in its killing powers; then, if a volley be fired from a regiment of 300 men once in every 4 minutes, and a regiment of artillery of 21 guns fire once in every 5 minutes; how many men will they kill together in 20 minutes?

(14) i. A square field contains 1417 acres 3318 yds. 3 ft. $10\frac{9}{16}$ in.; how many yards is it half round?

ii. Find the square root of ·0003250837848064; and the cube of 7507.

iii. A cubical block of stone contains $2326\frac{13}{64}$ c. ft.; find the area of its surface.

(15) i. Find the quantity of carpeting required for the central portion of a room, this portion being 14 ft. 5 in. wide, and 18 ft. 7 in. long. Also the cost, the carpet being $\frac{3}{4}$ yd. wide, and 4*s*. 6*d*. per yard.

ii. If in (i.) between the edge of the carpet and the walls there is a distance all round of $2\frac{1}{2}$ feet, how much of the area of the floor will remain uncovered? Also the cost, if it were inlaid with ornamental tiles at 3*s*. 5*d*. per square foot?

iii. A and B can perform a certain task in 30 days, working together. After 11 days, however, B is called off, and A finished it by himself 28 days after; how long would each take to do the work alone?

PAPER XXIV.

(1) i. Express $\frac{76}{729}$ and $\frac{89}{8101}$ with denominators 16767 and 575171 respectively; also find what number the numerator and denominator of the fraction $\frac{1219}{1404}$ must be multiplied by to produce $\frac{73140}{84240}$.

ii. Simplify $5\frac{2}{3}+3\frac{3}{5}-\frac{3}{4}-\frac{11}{12}+\frac{17}{20}-2\frac{5}{16}+13-5\frac{5}{6}-9$, and express the result in a decimal form.

iii. A man owns $\frac{10}{21}$ of $\dfrac{3\frac{3}{11} \text{ of } \frac{7}{18}}{\frac{7}{99} \text{ of } \frac{28}{29} \text{ of } 6\frac{3}{14}}$ of $\frac{1}{5}$ of a mine, and sells $\dfrac{5\frac{1}{5} \text{ of } \frac{7}{12} \text{ of } \frac{2}{13}}{\frac{3}{4} \text{ of } 5\frac{1}{2} \text{ of } \frac{2}{3}}$ of $\dfrac{14}{\frac{7}{12} \text{ of } \frac{21}{22}}$ of $\frac{1}{8}$ of his share for 3480*l.* 1*s.* 4*d.*; what portion of the whole has he left, and find the value of his original share, as well as of the whole mine?

(2) i. Express in the simplest form

$$\left(\frac{17\frac{1}{2}\times 3\frac{1}{7}\times 6}{3\frac{3}{4}-2\frac{5}{6}} \div \frac{13\frac{2}{11} \text{ of } \frac{22}{29}}{17\frac{1}{5} \text{ of } 1\frac{7}{43}}\right)+\frac{\frac{15}{31} \text{ of } \frac{7}{9} \text{ of } 15 \text{ of } 3\frac{4}{7}}{\frac{5}{93} \text{ of } 1\frac{10}{11} \text{ of } 3\frac{13}{14}}.$$

ii. How many sailors, each receiving $\frac{7}{10}$ of $\frac{11}{21}$ of $\dfrac{30\times 4\times 17}{51\times 9\times 20}$ of $7\frac{5}{7}$ of $\dfrac{10\frac{2}{3}}{3\frac{1}{5}}$ of 3 guineas, participated in the sharing of prize money to the amount of 10381*l.* 16*s.*?

iii. In what time can A, B, C, working together, build a wall, if A could do it alone in 12 days, A and B in 7 days, and B and C in 9 days?

(3) i. Find the number of cubic inches in 25 qrs. of corn, reckoning 277·274 c. in. to the imperial gallon.

ii. If ·00033904 be the quotient, and 250 the divisor, find the dividend, and express it as a vulgar fraction.

iii. How many boys are in a school of eight classes, the 1st containing ·0625, the 2nd $\cdot 08\dot{3}$, the 3rd, 4th, and 5th, each ·125, the 6th $\cdot\dot{1}$, the 7th $\cdot 1\dot{6}$ of the whole respectively, and the 8th 29 boys?

(4) i. Divide ·0091325 by 1562·5, and express $\cdot 09791\dot{6}$ as a vulgar fraction.

ii. On 6 successive days I spent the following portions of a legacy: $\cdot\dot{2}8571\dot{4}$, ·125, $\frac{1}{12}$, $\frac{1}{9}$, $\cdot 2\dot{1}4285\dot{7}$, and 91*l*., which was all that remained; how much did I spend each day, and what was the amount of the legacy?

iii. Find the difference in lbs. between 2·345 of a ton + ·0075 of a cwt. + ·5375 of a qr., and 1·2725 of 2 tons + ·625 of 5 cwt. + ·0125 of 3 qrs.

(5) i. Find the cost of glazing 115 windows, each containing 135 panes, at 4*s*. 3*d*. per dozen panes.

ii. A merchant paid income-tax on 59384*l*. at $9\frac{3}{4}$*d*. in the £; find his net income.

iii. Bought a hhd. of sugar containing 14 cwt. 2 qrs. 16 lbs., at 2*l*. 10*s*. per cwt.; 3 chests of tea, each 3 qrs. 11 lbs., at 3*s*. $7\frac{1}{4}$*d*. per lb.; and 15 cwt. 2 qrs. 9 lbs. of soap, at 30*l*. 6*s*. 8*d*. per ton; find the cost of the whole.

(6) i. What is the net weight of 67 hhds. of tobacco, weighing gross 237 cwt. 2 qrs. 24 lbs., tare $15\frac{3}{4}$ lbs. per cwt., and tret 4 lbs. in 104 lbs.?

ii. Find the value of 75 barrels of soda weighing gross 317 cwt. 2 qrs. 8 lbs., allowing 14 lbs. for the weight of each barrel, at $6\frac{1}{4}$*d*. per stone of 14 lbs.

iii. How many lbs. net are there in 70 casks weighing gross 624 cwt., tare 7 lbs. per cwt., tret 4 lbs. in 104 lbs., and draft 2 lbs. in 3 cwt.?

(7) i. On the 13th of August, 1863, I borrowed 375*l*. 12*s*. 6*d*., and on March 19, 1864, I repaid the loan, with interest at $7\frac{1}{2}$ per cent. per annum; how much did I pay?

ii. At what rate per cent. will the interest of 1024*l*. 17*s*. 6*d*. become 336*l*. 3*s*. $2\frac{4}{25}$*d*. in 6 years 16 weeks?

iii. What sum will 866*l*. 13*s*. 4*d*. amount to at Compound Interest in 4 yrs. at $6\frac{1}{4}$ per cent. per annum?

(8) i. Find the discount and present worth of 92*l*. 7*s*. $6\frac{3}{4}$*d*., due in 3 years 9 months, at $7\frac{1}{2}$ per cent.

ii. Find the difference between the bankers' and true discount of a Bill of 583*l*. 4*s*. $9\frac{1}{2}$*d*. due in 185 days, at 10 per cent.

iii. Find the present value of a Bill of 2291*l*. 18*s*., $\frac{1}{4}$ of which is due in 5 months, $\frac{5}{12}$ in 8 months, and the rest in 15 months, reckoning discount at 6 per cent. per annum.

(9) i. How much money must be invested in the 3 per cent. Consols when they are at $92\frac{1}{2}$, to produce the same income as would be produced by 1330*l.* invested in the $3\frac{1}{2}$ per Cents. at 95?

ii. What sum of money is necessary to pay $\frac{1}{8}$ per cent. commission and to purchase 5000*l.* Stock in the Funds, when they are at $92\frac{3}{4}$?

iii. A person invests 2582*l.* in railway shares at 115, the annual dividend on each share being 5*l.* He afterwards sells out at 125, and invests in the 3 per cent. Consols at 93; find the change in his income.

(10) i. A dealer buys goods for 1050*l.*, and sells after 6 months, for 867*l.*; how much did he lose, considering money to be worth 8 per cent. per annum?

ii. If a person transfers 1234*l.* from the $3\frac{1}{4}$ per Cents. at $88\frac{7}{8}$ to the $3\frac{1}{2}$ per Cents. at $95\frac{2}{3}$, find the difference in his income.

iii. In a subscription list one-half of the subscriptions are a guinea each, one-third a half-guinea each, and the 5*s.* subscriptions, which complete the list, amount to 693*l.*; find the whole amount subscribed, and the amount of the guinea and half-guinea subscriptions, and also the number in each class of subscribers.

(11) i. The number of votes polled in all by two candidates at an election was 13785, and the majority of the successful candidate was 527; what number of votes did each candidate receive?

ii. The express train leaves Cambridge at 9·22, and reaches London at 11, the distance being $57\frac{1}{2}$ miles; find the time required for a body travelling with the velocity of such a train to reach the sun, whose distance from the earth is 95 millions of miles.

iii. If the discount on a certain sum of money due 15 months hence be 6*l.* 4*s.*, find the amount; the rate being 5 per cent.

(12) i. A man sells as coffee a mixture of coffee and chicory at 1*s.* 9*d* per lb. If coffee be 2*s.* per lb. and chicory 6*d.*, and if he gain 5 per cent. on his outlay, determine the ratio in which he mixed the coffee and chicory.

ii. Simplify $$\frac{3 + \frac{5}{3} \text{ of } \dfrac{21}{7\frac{3}{5}} - \frac{1}{4} - \dfrac{1\frac{2}{3}}{2\frac{1}{2}}}{10 - \frac{151}{228} \text{ of } 5}$$

iii. Three partners in trade contribute respectively the sums of 438*l.*, 292*l.*, and 730*l.*, with the agreement that each was to receive 5 per cent on his investment, and that the remainder of the gains of the firm, if any, was to be divided between them in the proportion of the sums originally advanced. The whole gain of the firm was 200*l.*; what was each man's share?

(13) i. A bankrupt owes to one creditor 1000 guineas, to each of two others 500*l.*, and to each of three others 150*l.*; his assets are 600*l.*; how much does he pay in the pound, and how much does the first creditor receive?

ii. I put the sum of 150*l.* in my bankers' hands on January 1, 1856, with the understanding that the money was to accumulate at 3 per cent. Compound Interest. I called for the money on May 1, 1859; how much ought I to have received?

iii. A man has three vessels A, B, and C, the capacities of which are 1 gallon, 2 gallons, and 4 gallons, respectively; A is empty, B is full of water, and C full of wine. He fills A from B, replenishes B from C, and then empties A into C. When he has performed this operation twice, what will be the proportion of the wine in B to the water in C?

(14) i. What number multiplied by itself will produce 289·0996285849; and what will 3593·59 produce if multiplied by itself?

ii. A certain number cubed produces 19759·644267875; find the square of the number.

iii. Find the cube root of $714516\frac{123}{125}$; and of 805369·357418319.

(15) i. The square root of a number in the ordinary scale is equal to 11 in the scale whose radix is 7; find the first number.

ii. Show how to find the value of 4*l.* 5*s.* 6*d.* multiplied by 4*l.* 5*s.* 6*d.*, stating how, in this, concrete numbers can be multipliers.

iii. A steam engine of 10-horse power, working 6 days, of 6 hours a day, consumes 10 tons of coals; how many tons of coals will be consumed by a 3-horse power engine, working at the same work for 2 days, of 7 hours a day?

PAPER XXV.

(1) i. Find $\frac{7}{8}$ of $\frac{5}{6}$ of $\frac{16}{21}$ of that sum of money of which $\frac{3}{22}$ of $\dfrac{11\frac{1}{3} \text{ of } 2\frac{14}{15}}{13\frac{3}{5}}$ of $2\frac{4}{7}$ of $\frac{5}{6}$ is 135*l.* 3*s.* 9*d.*

ii. A class consists of 9 boys, whose respective ages are $13\frac{1}{2}$, $13\frac{1}{3}$, $13\frac{2}{3}$, two of $13\frac{5}{6}$, $14\frac{1}{4}$, $15\frac{1}{8}$, $16\frac{11}{12}$, and $18\frac{5}{8}$; find the average.

iii. What fraction of the least common multiple of 10, 14, 16, 18, 20, 24, 28 and 30 is the greatest common measure of 7920, 38160 and 241488?

(2) i. Reduce to their simplest terms $\left.\right\}$ $\dfrac{5\frac{5}{9} \text{ of } \frac{7}{18}}{\frac{3}{10} \text{ of } 7} \times \dfrac{3\frac{1}{11} \text{ of } 7\frac{1}{3} \text{ of } 8}{6\frac{1}{4} \text{ of } \frac{1}{14} \text{ of } 9}$; $\dfrac{2\frac{4}{5}-2\frac{1}{3}}{\frac{2}{3}+\frac{4}{5}} \div \dfrac{\frac{6}{7}}{5\frac{1}{7} \text{ of } \frac{5}{18} \text{ of } \frac{1}{6}}$.

ii. $\frac{3}{4}$ of gunpowder is nitre, $\frac{3}{20}$ charcoal, and the remainder is sulphur; how much of each ingredient will be required for $5\frac{1}{2}$ tons of powder?

iii. In a theatre $\frac{13}{16} - \frac{2}{3}$ of the audience were in the dress circle, $\dfrac{\frac{17}{18}}{4\frac{13}{18}}$ of $\frac{5}{8}$ were in the side boxes, $\frac{1}{16}+\frac{3}{20}+\frac{1}{24}$ were in the pit, $\dfrac{1\frac{4}{11} \text{ of } \frac{17}{35}}{\frac{5}{22} \text{ of } \frac{1}{12}}$ of $\frac{5}{7}$ of $\dfrac{\frac{20}{21}}{2\frac{1}{7} \text{ of } \frac{34}{49}}$ of $\frac{3}{128}$ were in the lower gallery, and 252 persons were in the upper gallery; find the number present in each part of the theatre, and the total.

(3) i. Express as decimals $\frac{4}{5}$ of $7\frac{1}{2} + \frac{11}{12}$; $5\frac{7}{8}$ of $3\frac{11}{64}$; $\dfrac{3\frac{1}{2} \text{ of } \frac{5}{8}}{2\frac{2}{11} \text{ of } \frac{11}{12}} \div \dfrac{2\frac{5}{8} \text{ of } 3\frac{1}{3}}{3\frac{1}{9} \text{ of } 7\frac{5}{7}}$.

ii. Find the continued product of ·0349, ·00785 and ·00747; also divide 27·3408 by ·00625, and find the difference between the two results.

iii. Add together ·35 of 13*s.* 4*d.*, $5\frac{5}{84}$ of a guinea, and $\cdot 91\dot{6}$ of 27·5 shillings; from this sum take $2\cdot 12\dot{0}3\dot{7}$ of a moidore + ·875 of a crown + ·625 of ·05*l.*, and express the result in positive terms.

(4) i. From $39\cdot\dot{9}2307\dot{6}$ subtract $11\cdot 0\dot{3}8461\dot{5}$; also divide $\cdot 3\dot{7}1428\dot{5}$ by 5·2.

ii. A owns $\cdot 10\dot{8}5714\dot{2}$ of a mill worth 22,225*l.*, B owns ·625 of the remainder; find the difference in value between their shares.

iii. A, B, C purchase a ship; A pays for $\cdot 6\dot{1}$ of it, B for $\cdot 208\dot{3}$ of it, and C the rest. They then take D into partnership, who buys $\cdot 1\dot{3}\dot{6}$ of A's share, ·8 of B's, and $\cdot 3\dot{4}6153\dot{8}$ of C's. What portion of the whole belongs to A, B, C, D respectively? And if D's share be worth 4365*l.*, find the value of A's, B's, and C's, and also the value of the ship.

(5) i. What does a creditor lose on a debt of 9559*l.* 16*s.* 8*d.* if he only receives 17*s.* 11¼*d.* in the £1?

ii. Find the value of 48 cwt. 2 qrs. 24 lbs. of metal at 11*s.* 10*d.* per cwt.

iii. A rents a farm of 795 acres 2 roods 38 poles, at 1*l.* 13*s.* 4*d.* per acre per annum; and one year his crop of potatoes, which he sold at 5 lbs. for a 1*d.*, cleared the rent, how many tons of potatoes had he?

(6) i. Find the net weight of 39 barrels of flour, weighing gross 160 cwt. 1 qr., allowing 16 lbs. per cwt. for the weight of the barrels, and 3 lbs. in 108 lbs. for tare.

ii. Find the net weight of 170 boxes of soap, each 3 cwt.

1 qr. 10 lbs., gross tare 17 lbs. per box, and tret 4 lbs. in 120 lbs.

iii. How many half ounces of tobacco can be weighed from 167 cases weighing gross 466 cwt. 2 qrs. 10 lbs., allowing 14 lbs. for the weight of each case, 4 lbs. in 104 lbs. for tret, and 2 lbs. in 300 lbs. for draft? And find the value of the same at 4½*d.* per oz.

(7) i. What sum lent at Simple Interest for 6 yrs. 8 wks. at 10⅜ per cent. will gain 223*l.* 18*s.* $7\frac{9}{25}$*d.*?

ii. In what time will 748*l.* 12*s.* 6*d.* amount to 1102*l.* 19*s.* 5⅘*d.* at 8 per cent. per annum?

iii. Compound Interest of 1866*l.* 13*s.* 4*d.* for 3½ yrs. at 5 per cent. per annum?

(8) i. Find the bankers' discount of a Bill of 627*l.* 10*s.* due in 292 days at 6¼ per cent.

ii. If on the 1st of May I took a Bill of 1020*l.* to a bank to be discounted, how much should I receive if the Bill was not due until September 21, the rate of discount being 7½ per cent. per annum?

iii. What will a banker gain by discounting a Bill of 3606*l.* 5*s.* 6¼*d.* drawn Oct. 3, at 9 months, and discounted Jan. 7, at 8 per cent. per annum?

(9) i. A person wishes to make an income of 100*l.* per annum by investing 2666*l.* 13*s.* 4*d.* in the 3 per cent. Consols; find what price they must be quoted at when he buys in.

ii. A person invests half his capital in the 3 per cents. at 91, and half in the 4 per cents. at par. Which investment produces the larger income? Supposing his total income to be 500*l.*, what is his capital?

iii. The 3 per cent. Consols are at 98, and certain railway securities are at £26 for original 20*l.* shares. What rate per cent. ought these last to pay, that it may be as profitable to invest in the one as in the other?

(10) i. A grocer buys tea at 5*s.* and 4*s.* per lb.; he mixes them, and then sells the mixture at 5*s.* per lb.; in what proportions are they mixed if he gains 20 per cent.?

ii. Archimedes discovered that the crown made for King Hiero consisted of gold and silver in the ratio of 2 : 1. How much per cent. was gold, and how much per cent. silver?

iii. The squire of a parish bequeaths a sum equal to $\frac{1}{100}$ of his estate towards the restoration of the church; 200*l.* less than this towards the endowment of the school, and 200*l.* less than the latter sum to the county hospital. After deducting these legacies $\frac{39}{40}$ of the estate remains to the heir; what was its value? And find the amount of each legacy.

(11) i. Sold flour at 15*s.* 9*d.* per cwt. and gained 8 per cent., how must it be sold to gain 24 per cent.? What weight must be sold at the latter price to gain 18*l.* 7*s.* 6*d.*?

ii. 100*l.* is put out to interest at 3 per cent., and accumulates for 125 years; at the end of that time the amount is invested in 3 per cent. Consols at 95; what will be the annual income therefrom?

[N.B.—$(1\cdot03)^{50} = 4\cdot3856$, and 4 places of decimals need only be retained throughout.]

iii. A viaduct consists of 3 series of arches built upon each other, the breadths of the arches in each being respectively 8 yds. 2 ft., 6 yds., and 5 yds.; whenever the piers in all the series are vertically above each other, there occurs a mass of masonry 4 yds. wide; of such there are 3. Find the length of the viaduct.

(12) i. How many bricks will be required for a wall 75 yds. long, 15 feet high, and 1 ft. 10½ inches thick, each brick being 9 inches long, ·375 of a foot broad, and 3 inches thick?

ii. Two cogged wheels work together, there being 32 cogs on one and 36 on the other. The larger wheel makes 64 revolutions in a second; how often will the same cogs be in contact in a working week of 10 hours each day?

iii. A is a working, B a sleeping partner in a business; A possesses 1200*l.* of the capital, B 2000*l.*: A receives

10 per cent. of all profits for managing, the rest being proportionately divided. Find the respective shares of 800*l.* profit.

(13) i. A and B run a race for 100 yds., and A wins by 17 yds., A and C run the same distance, and A wins by 8 yds.; how many yards' start ought C to give B that racing for 350 yds. they may make a dead heat (assuming A, B and C to run respectively at the same speed throughout)?

ii. Simplify $$\frac{\dfrac{1}{1-\dfrac{1}{2-\frac{1}{2}}}-\frac{1}{3}+(1+\frac{1}{2})}{3\left(1+\dfrac{2}{3\frac{1}{2}}\right)-4}.$$

iii. Sold rice at 15*s.* 6*d.* per cwt. and lost 7 per cent., what would have been the selling price if I had lost 14½ per cent.? How many bags, each containing 3 cwt. 1 qr. net, did I sell at the latter price if I sustained a total loss of 375*l.* 16*s.* 5¼*d.*?

(14) i. Find the square root of ·019 to eight figures; also the square of $\frac{473}{479}$.

ii. The square of a number is 6290082444·8016, find the number; also cube 50475.

iii. Extract the cube root of ·001117740530152; and of $\frac{28372625}{87528384}$.

(15) i. If 6 men can dig 14 yds. per day of a trench 3 ft. wide and 2 ft. deep, how many men will be required to dig 12 yds. in 1 day of a trench 7 ft. wide and 6 ft. deep?

ii. A plank 9 inches wide, 2 inches thick, and 8 feet long, weighs an oz. to the cubic inch; find the length and thickness of another plank of the same material, width and solid content, if 2 inches cut off from the end weighs 27 oz.

iii. If 18 cannon fire five rounds in 6 minutes, and kill 150 men in 20 minutes, how many cannon firing 6 rounds in 5 minutes will kill 360 men in 40 minutes?

The following formulæ for the solution of questions in mensuration, &c. may be useful:—

THE TRIANGLE.

(1) The area of a triangle is equal to half the product of the base and the perpendicular height.

(2) The area is equal to half the product of any two adjacent sides and the sine of the included angle.

(3) The area is equal to the square root of the continued product of half the *sum* of the sides and the three remainders arising from subtracting each side from half the sum of the sides.

(4) When one side only, and two angles of the triangle are known, the area may be found thus:—Multiply together the square of the known side, and the sine of each of the adjacent angles, and divide the continued product by twice the sine of the third angle.

If a, b, c be the sides, A, B, C the angles of a triangle, p the altitude, and s, half the sum of the sides.

The above four rules are expressed thus, in symbols—

(i.) Area $=\frac{1}{2}\, b\times p$ if b be the base.

(ii.) Area $=\frac{1}{2}\, a\, b \sin c$

(iii.) Area $=\sqrt{s\,(s-a)\,(s-b)\,(s-c)}$.

(iv.) Area $=\dfrac{a^2 \sin B \sin c}{2 \sin A}$.

THE PARALLELOGRAM (RECTANGULAR OR OBLIQUE).

(5) A parallelogram can always be divided into two triangles, and its area will be equal to twice the area of one of these triangles. Thus using the symbols.

i. Area $=p\times b$.

ii. Area $=a\, b \sin c$.

FOUR-SIDED FIGURES.

(6) A trapezium with two sides parallel; the area = half the product of the perpendicular distance between those

two sides, and their sum; or, if a and b be the sides, and p the perpendicular distance, the area $= \frac{p}{2}(a+b.)$

(7) If in any quadrilateral figure the diagonal be known and the perpendicular distances of the opposite angles from it, the area = half the product of the diagonal and the sum of the perpendicular distances.

Or in symbols, let d = diagonal, p_1, p_2 the perpendiculars, then area $= \frac{d}{2}(p_1 + p_2)$.

(8) If the sides and opposite angles be known, it can be divided into two triangles and found by (2).

THE CUBE AND PARALLELOPIPED.

(9) The volume of a cube is equal to the cube of one of its edges.

(10) The volume of a parallelopiped is equal to the continued product of the length, breadth, and height.

THE CIRCLE AND CIRCULAR RINGS.

(11) Let r be the radius, and π (the ratio of the circumference of the circle to its diameter) $= 3{\cdot}14159$, or $\frac{22}{7}$, or $\frac{355}{113}$.

The area $= \pi\, r^2$.

The circumference $= 2\, \pi\, r$.

The area of the sector $= \frac{1}{2}$ (arc × radius.)

(12) Let r_1, r_2 be the radii of the outer and inner circumferences of a circular ring.

The plane area $= \pi\,(r_1{}^2 - r_2{}^2)$.

(13) The area of a segment of a circle is equal to the difference between the area of the sector and the area of the triangle formed by the sides of the sector and base of the segment.

THE POLYGON.

(14) Let n be the number of sides of a regular polygon, a the length of each side, r the radius of the inscribed circle, and R the radius of the circumscribed circle.

i. The area $=\frac{n\,a^2}{4}\cot.\frac{180^\circ}{n}$.

ii. „ „ $=n\,r^2\tan.\frac{180^\circ}{n}$.

iii. „ „ $=\frac{1}{2}\,n\,R^2\sin.\frac{360^\circ}{n}$.

iv. Also, $r=\frac{a}{2}\cot.\frac{180^\circ}{n}$.

$$R=\frac{a}{2\sin.\frac{180^\circ}{n}}$$

THE PRISM, CYLINDER, CONE, AND PYRAMID.

(15) Let p be the perimeter of the end of the prism or cylinder, or of the base of the cone or pyramid, h the height or length, l the length of the slant side of the cone, and A^2 the area of the end, or base of each.

i. The volume of the prism, or cylinder $=h\,A^2$.
ii. The convex surface $=p\,h$.
iii. The volume of the cone or pyramid $=\frac{1}{3}\,h\,A^2$.
iv. The convex surface of the cone $=\frac{1}{2}\,p\,l$.

(16) For the frustum of a cone or pyramid, let $p_1\,p_2$ be the perimeters of the ends, $A_1{}^2\;A_2{}^2$; their areas, h_1; the height of the frustum, l_1 its slant height.

i. The volume of the frustum $=\frac{h_1}{3}(A_1{}^2+A_1\,A_2+A_2{}^2)$.

ii. The surface of the frustum $=\frac{l_1}{2}(p_1+p_2)$.

THE SPHERE.

(17) Let r be the radius of the sphere.
i. The volume $=\frac{4}{3}\,\pi\,r^3$.
ii. The surface $=4\,\pi\,r^2$.

(18) For the segment of a sphere, let h be the height, ρ the radius of the base, and r the radius of the sphere.

i. Volume of the segment $= \frac{\pi}{6} (6\,r - 2\,h)\,h^2$.

or, $= \frac{\pi}{6} (3\,\rho^2 + h^2)\,h$.

ii. Convex surface $= 2\,\pi\,r\,h$.

ARTIFICERS' WORK.

(19) The standard thickness of brickwork is three half bricks. Any thickness may be reduced to this by multiplying the superficial area of the wall, &c., by the number of half bricks in the thickness, and dividing the product by 3.

N.B.—The standard rod of brickwork contains 272¼ square feet.

MEASUREMENT OF SHOT, SHELLS, POWDER, ETC.

(20) i. An iron ball 4 inches diameter, weighs 9 lbs.
ii. A leaden ball 1 inch diameter, „ $\frac{3}{14}$ lb.
iii. 30 cubic inches of powder weigh 1 lb.

PILES OF BALLS AND SHELLS.

(21) If n be the number of balls in a side of the base row, and l be the number of balls in the longer side of the base.

i. The number of balls in a triangular pile $= \frac{1}{6}\,n\,(n+1)\,(n+2)$.

ii. square „ $= \frac{1}{6}\,n\,(n+1)\,(2\,n+1)$.

iii. rectangular „ $= \frac{1}{6}\,n\,(n+1)\,(3\,l - n + 1)$.

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